

# Research into the loss of synchronism in power systems due to disturbances based on an analysis of electromechanical waves

N.N. Lizalek, V.V. Vasiliev\*

Novosibirsk State Technical University, Novosibirsk, Russia

**Abstract** – In this paper, we suggest studying the stability of power systems under disturbances by analyzing the processes of synchronism loss, given electromechanical waves in power system that form an oscillatory structure of motion. In contrast to the classical formulation of the stability problem, an indispensable part of the study on the loss of synchronism is the location of an out-of-step cutset in the system (the spatial structure of instability). The problem of synchronism loss analysis has two statements: the prediction of possible instability structures and the determination of the instability structures in specific emergencies. The prediction of instability structures is made based on the equal areas method applied to the motion of excited oscillators in the system. In this case, possible instability structures are selected by comparing the energy and time characteristics of the unstable motion. The analysis of the processes of synchronism loss in power systems for a given emergency involves the calculation of the kinetic energy of mutual oscillations of an unstable pair of an oscillatory structure. The identification of the unstable pair of subsystems enables the out-of-step cutset to be located. The excess kinetic energy of the mutual motion of the unstable pair subsystems becomes an energy characteristic of the loss of synchronism.

**Index Terms** – Power systems, electromechanical waves, oscillatory structures, oscillators of the system, possible and actual trajectories of motion, energy-time diagrams, unstable pair

---

\* Corresponding author.  
E-mail: [v\\_v\\_vasiliev@mail.ru](mailto:v_v_vasiliev@mail.ru)

<http://dx.doi.org/10.25729/esr.2018.02.0008>

Received: July 06, 2018. Revised: August 27, 2018.

Accepted: October 02, 2018, Available online: October 15, 2018.

© 2018 ESI SB RAS and authors. All rights reserved.

## I. INTRODUCTION

The complexity of electromechanical transients in power systems manifests itself in the complexity and diversity of the processes related to upsetting the stability of parallel operation of synchronous machines (angle stability).

One of the important goals of the practical study on the loss of stability is to determine the position of the out-of-step cutsets under various emergency disturbances. Identification of the *spatial structure of instability* becomes particularly relevant when building automatic emergency control systems to ensure the stability of complex power systems under emergencies that occur far from generating sources.

Classical methods of studying the stability of complex power systems (the method of "small" oscillations, transient stability analysis using the Lyapunov function) do not determine the position of an out-of-step cutset [1-4]. The studies are completed as soon as the *fact of stability or instability* of the considered system is established. The out-of-step cutset is detected beyond the stability study procedures, after the numerical calculation of the unsteady transient process on the basis of an analysis of changes in the angular coordinates at different nodes of the system over time.

Practical analysis of stability requires an answer not only to the question – ‘Will there be a stability loss in case of a particular disturbance (finite or "infinitesimal")?’, but also to the questions: ‘In what cutset will this loss of stability occur and how does its spatial position depend on the properties of the system and disturbance?’ Within the framework of such an extended formulation of stability problems, the issue of the spatial position of the cutset where the out-of-step conditions develop (the structure of unstable motion) becomes an integral part of the methods and algorithms of the study. Such a statement can be called the problem of analysis of disintegration processes of synchronous operation (synchronism) of power systems under disturbances (the structural stability analysis).

The issue of a structure of unstable motion is part of the more general issue of the spatial structure of electromechanical motion in a complex system. The spatial structure of motion (hereinafter referred to as the structure of motion) can be understood as the division of a system into regions (subsystems), within which the motion of

components included in them at the time under consideration have some common qualitative feature. It is worth noting that this common feature should be determined not only for the rotors of synchronous machines but also for the voltage vectors at the nodes of the system. This makes it possible:

- to cover the entire space of the power system;
- to determine which pairs of connected nodes belong to different subsystems, i.e. to distinguish the ties between them, by determining the boundaries of the subsystems and their connectivity with others and, consequently, to build a topology of motion in the system.

Such a feature can be represented, for example, by the *sign of angle change* (between the voltage vector or the longitudinal axis of the synchronous machine rotor and the vector rotating at a speed of the center of the system inertia), which occurred from the emergency disturbance to the considered time.

The idea of the center of inertia of the power system was introduced in [5] and repeatedly used [6,7, etc.]. The velocity and motion of the center of inertia of the power system describe its general motion under the influence of disturbances. The characteristics of the motion of the inertia centers of the subsystems determined in one way or another were not used in the studies of transients and stability. A remarkable property of non-inertial systems of coordinates associated with the centers of inertia of the system and subsystems (zero total momentum of motions relative to the centers of inertia [8]), makes it possible to significantly simplify the calculations of kinetic and potential energy, determine their spatial distribution and qualitative composition (regional and local components) and, on this basis, analyze the processes of loss of synchronism.

The structure of motion is determined by many factors: the network structure of the system and the "rigidity" of connections, the distribution of inertial masses, the action of control systems, the location of the disturbance and its severity. It is an important characteristic of the transient process, whose development in time and space results (under stability loss) in the spatial structure of instability.

The processes of stability loss are closely related to the physical effects observed during electromechanical oscillations in the power system. These effects manifest themselves in the *spatial patterns* of the network (in the nodal space) of the power system. Computational studies allow us to establish five qualitative effects because power systems belong to the class of distributed oscillatory systems (in the format: effect - to the left, its cause – to the right):

1. A small number of free oscillations observed under any specific disturbances.

A small number of observed oscillations under disturbances in an extended system.	=	The resonance nature of the system response determined by the place of disturbance.
---	---	---

2. The phenomenon of oscillation dispersion caused by the concentration of rotating masses.

The farther the observation area from the emergency center, the later and slower the oscillations begin in it.	=	Low-frequency electromechanical oscillations propagate faster than high-frequency oscillations.
--	---	---

3. The wider spread of low-frequency oscillations.

The lower the frequency of oscillations, the greater part of the system experience them.	=	Low-frequency oscillations have longer wavelengths and greater penetration than high-frequency oscillations.
--	---	--

4. The oscillations in the system near its center of inertia occur in the form of motions of areas (subsystems), any adjacent of which move in opposite directions, i.e. the oscillatory motion is *wavelike distributed* throughout the system. As the transient process develops, the number and composition of subsystems, as well as the method of their integration by inter-regional ties, i.e. the structure (topology) of the observed oscillatory motion (its oscillatory structure), in general, change. This effect can be represented by the ratio:

Oscillations occur in the form of oppositely directed motions of adjacent areas (regions) of the system.	=	The topology of oscillatory motions is determined by their wave character.
--	---	--

5. The fifth qualitative effect is associated with a *possible change in the spatial position of the cutset* with primarily developing out-of-step conditions under a further increase in the disturbance severity, which leads to stability loss. This effect is naturally associated with the changes in the relationships between the conditions of the motion development along the limit (in terms of stability) paths in the extended power system, which contains a lot of *weak links* that appear in different places. These weak links manifest themselves through the development of instability under different values of the limit disturbance in the considered place of the system at different times. It appears that the observed structure of the primarily developing out-of-step condition is determined by the location of the weak link, which reaches the critical stability state first. This consideration can be represented as follows:

A change in the instability structure with a change in the disturbance severity.	=	A change in the location of the weak link that reaches the critical state first.
--	---	--

The basic idea of the structural stability analysis is the assumption that the instability is always associated with "weak" cutsets in the system, whose overload at dynamic or static variations in the conditions leads to instability. The manifestations of these weak links in the processes of

stability loss can be associated with energy characteristics of the electromechanical transient process: its kinetic and potential energy and *the spatial distribution of these components* in the system.

Changes in the kinetic and potential energy of the system and its parts are determined by the trajectory of motion. When choosing the trajectories of the system motion, studied for stability, it is advisable to use trajectories, obviously dangerous for stability. Due to the fourth of the above-mentioned properties of electromechanical oscillations, it can be assumed that these trajectories should be chosen on the basis of studies of the structure of the oscillatory motion developing in the system under emergency.

The method used to study electromechanical processes and stability of power systems is based on an analysis of spatial-temporal characteristics of the system response to disturbing effects caused by wave processes in a distributed oscillatory system with lumped masses [9, 20]. The wave approach has been recently widely used to describe the motion of energy systems and to state the stability problems. However, despite the development of microprocessors and technologies for obtaining phasor measurements [10-12], the algorithms for emergency control to prevent stability loss in power systems [13] do not use the wave approach yet.

In the known studies devoted to the development of the wave approach [14-18], there are features that do not enable them to be fully used in the analysis of the processes related to the loss of synchronism. Firstly, the models with distributed mass are often used, which, generates an unlimited spectrum of frequencies and destroys dispersion when considering wave processes. Secondly, the concept of the structure of motion is not used, which significantly complicates the formulation of stability problems of complex power systems and solving them.

Thus, the kinetic energy and potential energy of the electromechanical transient process, defined in [17,18], are not associated with the structure of oscillatory motion, their distribution in the space of the power system and the effect of this distribution on stability are not considered. At the same time, the concept of the structure of motion is quite natural within the wave approach, which has the ability to operate positive and negative half-wave areas of the system. Thirdly, electromechanical waves are considered to analyze the propagation of disturbances in the running version, which makes it difficult to study the weak links of the system. Weak links of the system are easier to identify on the basis of standing waves. In addition, in general, these studies are intended for a mathematical description of electromechanical motion and instability processes, their physical content recedes into the background.

The application of the wave approach used in conjunction with the analysis of the structure of motion for the study of the processes of synchronism loss is the main content of this paper. The wave approach used to identify the

structure of motion allows us to consider the processes from a physical point of view: namely, the loss of synchronism of the power system as a combination of the wave process of the oscillatory structure formation and the development of unstable motion between the arising objects of this oscillatory structure.

In [19-25], the authors present the main content of the approach used to study the processes of loss of synchronism in complex power systems. These studies are focused on:

- the structures of "small" standing electromechanical self-oscillations of extended power systems of different scale (hereinafter referred to as wave structures) [20];

- the algorithms for estimation of transient stability with the use of wave and oscillatory structures, based on energy relations (the equal areas method for the oscillatory structures of motion) [19,21,23,24];

- the statements and proposals for solving the problems of predicting the out-of-step conditions due to disturbances in the power system and identifying the out-of-step conditions of generators (the center of the swing is inside the synchronous machine) [22];

- the algorithms for identification of instability and location of an out-of-step cutset using the results of the integration of the equations of the mathematical model of the power system [25];

- the algorithms for selecting control actions of emergency control systems to ensure stability, using dynamic models of the control object [25].

The equal areas method is the most well-known and long-used method of dynamic stability analysis [1,2,26, etc.]. This method, used in this paper for oscillatory motion structures, is designed to search for limit transient disturbances that lead to the loss of transient stability between the subsystems of the oscillatory structure in the first and second cycles of swings. It is implemented by calculating [23, 24]:

- the relative velocities of motion of regional subsystems of the oscillatory structure after any disturbance (emergency scheme);

- the ascending branches of trajectories of regional displacement of subsystems of the oscillatory structure (with increasing mutual angles between the centers of inertia of the adjacent subsystems) in the pre-emergency scheme (coinciding with the post-emergency one) in the first and second cycles of swings (conditions on the ascending branch of the first cycle are determined by a staged change in the generator angles proportional to the relative velocities of their regional motion, the conditions of the second cycle of swings are calculated similarly for the opposite signs of relative velocities);

- the full (for zero duration of emergency) braking margins for the subsystems in the first and second cycles of swings until they reach the maximum displacement identified by the appearance of a positive time derivative of the total kinetic energy of regional motion;

- the acceleration work of subsystems in the emergency scheme by calculating a series of conditions under changes in the generator angles proportional to their regional relative velocities;

- the conditions (full kinetic energy of oscillations, acquired by the system in emergency conditions, the maximum duration of the emergency, structure of instability) under which the acceleration work of the subsystems can be compensated by the remaining braking work in the first cycle or braking margin in the second cycle of swings.

As in the classical equal areas method, modeling of the power system is simplified (synchronous machines are represented by constant EMF behind transient reactance, the differential equations of rotor motion do not allow for the damping torque). The load can be represented by shunts and static voltage characteristics. All calculations are carried out for the complete scheme of the power system.

Below we consider two statements for the nonlinear problem of studying the processes of the loss of synchronism in an electric power system:

1. *Forecast of possible options of the synchronism loss* for some set of disturbances to make a general overview of the stability problems.

2. *Identification of the synchronism loss* during the transient process and determination of its characteristics under specific topology and operating conditions.

The first of them is based on the method of equal areas for oscillatory motion structures, does not require calculations of electromechanical transients and is designed to:

- estimate the conditions for the development of unstable motion (determination of limit disturbances);
- determine instability structures at disturbances in different places of the power system;
- identify actual cases of changes in the instability structure with an increase in the emergency severity;
- estimate time characteristics of unstable processes to formulate the requirements for emergency control systems;
- determine the requirements for the speed of relay protection and switching devices.

The second solution can be used to:

- construct automatic emergency control systems that employ a *dynamic mathematical model of the system of any complexity* when selecting control actions to ensure the stability of complex power systems;
- visualize the power system modeling results (to display the motion of the emerging subsystems) by computer software for the calculation of electromechanical transients;
- identify the structure of the power system (or its part) motion based on the phasor measurements used to control it.

The results of the research are presented in Sections 2, 3 and 4 of the paper. In Section 2, we propose a hierarchical structure of coordinate systems for describing the motion of synchronous machines. The power and energy characteristics of the motion of the power system divided into

subsystems in the coordinate systems associated with the center of inertia of the system and the centers of inertia of the subsystems are determined. The idea of system-wide, regional and local processes is introduced. An emphasis is placed on the independence of the system-wide energy characteristics of motion and the total energy of oscillations from the assumed subdivision into subsystems.

Section 3 presents the basic concepts and algorithms to identify the instability structures in the case of transient disturbances of the considered steady-state conditions in a power system give an overview of the composition of the stability problems. Since the calculations are of an assessment nature (carried out without integration of the equations of the mathematical model of the system), the structures of the motion coincide with the wave (oscillating) structures of "small" oscillations excited in case of an emergency in the considered place of the system. In this case, it is assumed that the "weak" ties of the system are sufficiently fully represented by the intersystem cutsets of wave structures describing "small" standing electromechanical waves of different frequencies in the power system with "off" damping.

Section 4 presents algorithms for tracking the development of the oscillatory structure of motion when integrating the equations of the mathematical model of the system used, a method for locating the cutset with out-of-step conditions and the energy characteristic of instability, i.e. the excess kinetic energy of an unstable pair of subsystems.

## II. THE STRUCTURE OF THE MOTION AND ITS POWER AND ENERGY CHARACTERISTICS

We introduce a formal representation of the *structurally organized motion* of power systems. Let us divide a set of nodes of the power system into a number of subsets. The nodes included in one of these subsets will be assigned to some subsystem. We will divide the system into subsystems so that each node of the system is included in only one subsystem, and, in addition, let all nodes of the subsystem form a simply-connected region on a network graph of the system. The system links that connect the subsystem nodes with each other will be called the internal ties of the subsystem. The links connecting the nodes from different subsystems will be referred to as intersystem ties. The resulting partition of the system into subsystems and intersystem ties connecting them represents a structural presentation (**structural model**) of the system.

To simplify, the structural model will be called the "structure" of the system  $S$  (which must be distinguished from the usual network structure). The number of subsystems included in the structure gives its dimension  $R(S)$ . It is clear that one and the same system can be represented by a set of its structural presentations (structures).

Let the motion of the system be known, i.e. all coordinates of the system are known as functions of time.

Let the rotation speed of the  $i$ -th synchronous machine be represented by a sum of constant component (rotation frequency in the initial conditions) and *three relative processes*:

$$\Omega_{gi}(t) = \Omega_0 + \Delta\Omega_{gis}(t) + \Delta\Omega_{s0}(t) + \Delta\Omega_0(t), \quad (1)$$

where:

$$\Omega_0 = \Omega_0(0),$$

$$\Delta\Omega_0(t) = \Omega_0(t) - \Omega_0,$$

$$\Delta\Omega_{s0}(t) = \Omega_{s0}(t) - \Omega_0(t),$$

$$\Delta\Omega_{gis}(t) = \Omega_{gi}(t) - \Omega_{s0}(t),$$

$$\Omega_0(t) = \frac{\sum_i J_i \Omega_{gi}(t)}{\sum_i J_i}, \quad \Omega_{s0}(t) = \frac{\sum_{i_s} J_i \Omega_{gi}(t)}{\sum_{i_s} J_i}.$$

The following symbols are introduced here:  $\Omega_0(t)$  - the velocity of the center of inertia of the system,  $\Omega_{s0}(t)$  - the velocity of the center of inertia of the subsystem to which this synchronous machine is assigned,  $i, i_s$  - the set of active (generator) nodes in the whole system and in the  $s$ -th subsystem,  $J_i$  - moment of inertia of the  $i$ -th synchronous machine,  $\Delta\Omega_0(t)$  - the deviation of the velocity of the center of inertia of the system at time  $t$  from the initial one in the steady state  $\Omega_0$  (the highest level of the motion hierarchy),  $\Delta\Omega_{s0}(t)$  - synchronous motion of the subsystem - *regional process*, determined by the deviation of the velocity of the center of inertia of the subsystem relative to the velocity of the center of inertia of the system (the average level of the hierarchy of motion),  $\Delta\Omega_{gis}(t)$  - individual motion - *the local process* of motion of a synchronous machine in the  $s$ -th subsystem relative to its center of inertia (the lowest level of the motion hierarchy).

The motion of the system, described by the introduced three-stage hierarchical system of relative processes, will be called *structurally organized*. The structural organization of the system leads to the allocation of objects at different levels: the system as a whole, subsystems and individual synchronous machines. Depending on the system structure, the same motion of the system will be structurally organized in different ways (will have different forms of structural organization). As can be seen from the above relations, the component of the motion of the center of inertia of the system remains unchanged with the variation in the system structure.

It is possible to represent the absolute motion of rotor of the synchronous machine  $\delta_{gi}(t)$  at time  $t$  as a sum of components

$$\begin{aligned} \delta_{gi}(t) &= \delta_{gi}(0) + \Omega_0 t + \int_0^t \Delta\Omega_0(t) dt + \int_0^t \Delta\Omega_{s0}(t) dt + \int_0^t \Delta\Omega_{gis}(t) dt = \\ &= \delta_{gi}(0) + \delta_0(t) + \Delta\delta_0(t) + \Delta\delta_{s0}(t) + \Delta\delta_{gis}(t), \end{aligned} \quad (2)$$

where  $\delta_{gi}(0)$  - initial value,  $\delta_0(t)$  - the displacement of the center of inertia of the system at a constant velocity of its motion,  $\Delta\delta_0(t)$  - the relative motion of the center of inertia of the system due to changes in the velocity of its motion in the interval  $(0-t)$ ,  $\Delta\delta_{s0}(t)$  - the angular displacement of the center of inertia of the subsystem relative to the center of inertia of the system,  $\Delta\delta_{gis}(t)$  - the angular displacement of the synchronous machine in the  $s$ -th subsystem relative to the center of inertia of the same subsystem.

It follows from the introduced definitions that if each generator of the system is assigned to a particular (but only one) subsystem, then, regardless of the method of splitting the system into subsystems, the total momentums of relative (near the centers of mass) motions of subsystems in the system and synchronous machines in subsystems are zero:

$$\sum_s J_s \Delta\Omega_{s0}(t) = 0, \quad \sum_{i_s} J_i \Delta\Omega_{gis}(t) = 0 \quad \text{for all } s. \quad (3)$$

where  $J_s = \sum_{i_s} J_i$  - total moment of inertia of the subsystem.

Differentiation and integration of (3) give the corresponding relations for relative accelerations and relative angular displacements

$$\sum_s J_s \frac{d(\Delta\Omega_{s0})}{dt} = 0, \quad \sum_{i_s} J_i \frac{d(\Delta\Omega_{gis})}{dt} = 0, \quad (4)$$

$$\sum_s \int_0^t J_s \Delta\Omega_{s0}(t) dt = \sum_s J_s \Delta\delta_{s0}(t) = 0, \quad (5)$$

$$\sum_{i_s} \int_0^t J_i \Delta\Omega_{gis}(t) dt = \sum_{i_s} J_i \Delta\delta_{gis}(t) = 0.$$

Taking into account the relations for the total momentums in the coordinate systems of the centers of inertia of systems and subsystems allows us to obtain the equations of motion of individual objects of the structure. The motion of the center of inertia of the system satisfies the equation

$$J_E \frac{d(\Delta\Omega_0)}{dt} = \Delta M_E, \quad (6)$$

where  $\Delta M_E$  is the total excess torque on the shafts of machines in the system.

The equation of relative motion of the center of inertia of the  $s$ -th subsystem in a non-inertial system associated with the center of inertia of the system can be written as follows

$$J_s \frac{d(\Delta\Omega_{s0})}{dt} = \Delta M_s - \frac{J_s}{J_E} \Delta M_E \quad (7)$$

The value  $\Delta M_{s0} = \Delta M_s - \frac{J_s}{J_E} \Delta M_E$  is an excess torque acting on the subsystem at its relative motion near the center of inertia of the system. It follows from the definition of  $\Delta M_{s0}$  that  $\sum_s \Delta M_{s0} = \mathbf{0}$ . The torques  $\Delta M_s$  act on subsystems in their absolute motion.

The equation of motion of the synchronous machine relative to the center of inertia of the subsystem, in which it is included, is

$$J_i \frac{d(\Delta \Omega_{gis})}{dt} = \Delta M_i - \frac{J_i}{J_s} \Delta M_s \quad (8)$$

The value  $\Delta M_{is} = \Delta M_i - \frac{J_i}{J_s} \Delta M_s$  is an excess torque acting on the synchronous machine at its relative motion near the center of inertia of the subsystem the machine belongs to. For the sums  $\Delta M_{is}$  and  $\Delta M_i$  for all machines in the subsystem, the ratios are performed  $\sum_{i_s} \Delta M_{is} = 0, \sum_{i_s} \Delta M_i = \Delta M_s$ .

The excess torques  $\Delta M_i$  affect the synchronous machine in their absolute motion.

The total kinetic energy of the system  $K(t)$  at time  $t$  is the kinetic energy in the steady state  $K_0$  plus additional kinetic energy  $\Delta K(t)$  gained during the transient period  $(0 - t)$ , i.e.  $K(t) = K_0 + \Delta K(t)$ . The additional kinetic energy of the system  $\Delta K(t)$  consists of the contributions of individual synchronous machines  $\Delta K_i(t)$ :  $\Delta K(t) = \sum_i \Delta K_i(t)$ .

When summing (given the relations (3)), we obtain the kinetic energy of the system in a transient process, expressed through the variables that define the structurally organized motion (9):

$$K(t) = K_0 + \sum_s \sum_{i_s} J_i \frac{\Delta \Omega_{gis}^2(t)}{2} + \sum_s J_s \frac{\Delta \Omega_{s0}^2(t)}{2} + J_E \frac{\Delta \Omega_0^2(t)}{2} + \Omega_0 J_E \Delta \Omega_0(t)$$

The additional total kinetic energy  $\Delta K(t)$  is divided into components (10):

$$\Delta K(t) = \sum_s K_{locs}(t) + \sum_s K_{regs}(t) + K_{sys}(t) = K_{loc}(t) + K_{reg}(t) + K_{sys}(t),$$

where  $K_{loc}$ ,  $K_{reg}$ ,  $K_{sys}$  - kinetic energies of *local* oscillatory processes in subsystems, *regional* oscillatory processes in the system and general motion in the *translational degree of freedom*. The last component is represented by the expression

$$K_{sys}(t) = \Omega_0 J_E \Delta \Omega_0(t) + J_E \frac{\Delta \Omega_0^2(t)}{2}. \quad (11)$$

The component  $K_{locs}$  characterizes the intensity of internal motions in subsystems, their "heating". The component  $K_{regs}$  determines the proportion of the kinetic energy of oscillations sent to the oscillatory motion of a region, considered as a whole.

The components  $K_{loc}$ ,  $K_{reg}$  characterize the processes due to the *oscillatory degrees of freedom*, they equal zero at synchronous motion in the entire system. The sum of the components

$$K_{osc} = K_{reg} + K_{loc} = \sum_s J_s \frac{\Delta \Omega_{s0}^2(t)}{2} + \sum_s \sum_{i_s} J_i \frac{\Delta \Omega_{gis}^2(t)}{2} \quad (12)$$

determines the total *kinetic energy* of the system oscillations. The latter relationship can be interpreted as a *spatial decomposition* of the kinetic energy of oscillations.

Since the total kinetic energy of the system in a transient process and its component determined by translational motion, do not depend on the structure of the system, the sum  $K_{osc} = K_{loc} + K_{reg}$  does not depend on the partition of the system into subsystems either. In the event of variation in the structure of the system, only the relationship between the kinetic energy of local and regional processes  $K_{loc}$   $K_{reg}$  will change.

Based on the equations for relative motion, we write the following relationships between the work and the changes in kinetic energy for the objects (system -subsystem- synchronous machine)

$$\int_{t_0}^t d \left( J_E \frac{\Delta \Omega_0^2}{2} \right) = \int_{\Delta \delta_0(t_0)}^{\Delta \delta_0(t)} \Delta M_E d(\Delta \delta_0), \quad (13)$$

$$\int_{t_0}^t d \left( J_s \frac{\Delta \Omega_{s0}^2}{2} \right) = \int_{\Delta \delta_{s0}(t_0)}^{\Delta \delta_{s0}(t)} \Delta M_{s0} d(\Delta \delta_{s0}), \quad (14)$$

$$\int_{t_0}^t d \left( J_i \frac{\Delta \Omega_{gis}^2}{2} \right) = \int_{\Delta \delta_{gis}(t_0)}^{\Delta \delta_{gis}(t)} \Delta M_{is} d(\Delta \delta_{gis}). \quad (15)$$

The right-hand parts of relationships (13 – 15) determine the work performed during the motion of the system objects. It can be associated with changes in the potential energy on their trajectories.

Similar relationships can be obtained with the introduction of a four-step hierarchy of motions with the selection of objects: system-platforms (zones) - subsystems-synchronous machines.

III. THE FORECAST OF THE SYNCHRONISM LOSS OPTIONS UNDER SHORT-TERM TRANSIENT DISTURBANCES

Possible instability development in different cutsets of a power system even with the unchanged location of the emergency center determines the necessity of introducing a concept of a spectrum of *limit disturbances of dynamic stability*. This is an energy spectrum. To determine the spectrum of limit disturbances means to estimate the minimum energy characteristic of the transient process accompanied by stability loss, in which case each structure of the excited instability will correspond to its energy index.

The processes of stability loss of the mutual motion on small time intervals (in the first and second cycles of swings) are essentially connected with the first, fourth and fifth of the above-mentioned properties of the power system as a distributed oscillatory system. The second and third properties on these time intervals do not have time to manifest themselves. They will begin to play a significant role in the processes of stability loss at more distant time instants (in the third, fourth, etc. cycles of swings).

The first property determines *the composition of the excited oscillators of the system* [19, 21] under considered disturbance, while the fourth and fifth properties jointly identify a system object winning the "race" to the loss of synchronism. A comparison of the instability options involves the determination and comparison of the following indices:

- the magnitude of the limit disturbance in terms of stability (the energy characteristics of limit disturbances);
- the time of the unstable motion development;
- the spatial configuration of instability (locations of out-of-step cutsets).

Figure 1 shows typical *small-size oscillating structures* taken from low-, medium - and high-frequency parts of the spectrum of "small" electromechanical self-oscillations of power systems [20] (they were obtained considering power systems of various sizes with the number of synchronous machines varying from units to hundreds). The numbered circles denote inphase moving groups of synchronous machines-subsystems, the straight lines stand for dynamic cutsets. For the oscillating structures, the inertia of subsystems is demonstrated by the change in the radius of a circle symbolizing the subsystem, proportionally to the cubic root of its total rotating mass.

The small-size oscillators of the system are its "weak" links represented by intersystem cutsets. Their manifestations through the processes of loss of stability under specific disturbances are determined by modeling the oscillatory structures of the motion (models of motion) developing in each of them. The identification of subsystems of model oscillatory structures on a particular oscillator under a specific disturbance in the system [19, 21] has a simple physical meaning: a subsystem is a connected region of the

system in which all the subsystems of the oscillating structure move in one direction relative to the center of inertia of the system.

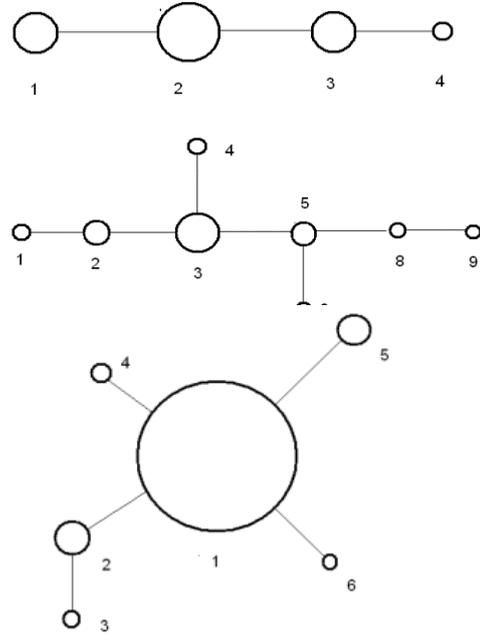


Figure 1. Examples of oscillating structures of power interconnections.

During its motion, the system is divided into subsystems maximum possible in terms of size within which the maximum attainable synchronism is provided. Opposite displacement of adjustment subsystems of the oscillatory structure is a precondition for instability of their mutual oscillations as whole structural entities. The maximum sizes of the subsystems mean that the study on the processes of loss of the intersystem stability of mutual motion in a given model oscillatory structure does not need a more detailed structural representation of the system by a larger number of subsystems.

Model oscillatory structures are distinguished on oscillating structures of power systems coinciding with the wave structures of "small" oscillations of different frequencies [20]. The oscillation frequency can be used as the "name" of the oscillator [19, 21]. The consideration of the first of the power system properties as a distributed oscillatory system is reduced to the use of oscillators associated with the regional spectra of small oscillations. This is achieved by specifying the support nodes of the application of the forcing harmonic action in the part of the system where the considered emergencies occur [20-22]. The model oscillatory structures constructed on the system oscillators characterizing the structures of "small" oscillations near the equilibrium point in the pre-emergency conditions can be used to describe the free motion of the system after the elimination of the disturbing effect.

The determination of the stability limit disturbances requires the calculation of the work under mutual oscillations in the system. The equations of energy balance at oscillations of objects of various hierarchical levels on time interval  $(t_0 - t)$  have the form (13 – 15). Integrals of (13-15) must be calculated on the trajectories of the system, i.e. at angular coordinates, velocities, and voltages that satisfy the equations of transient process of the system. Let all coordinates and corresponding velocities of relative motion of all objects of the system (subsystems and individual synchronous machines within the subsystems), and, consequently, all unbalances of moments, be known at some time  $t_0$ . The differentials of the system variables at any time are determined by the equations of motion.

$$d(\Delta\delta_{s0}) = dt \cdot \Delta\Omega_{s0}(t), \tag{16}$$

$$d(\Delta\delta_{gis}) = dt \cdot \Delta\Omega_{gis}(t),$$

$$d(\Delta\Omega_{s0}) = \frac{dt}{J_s} \Delta M_{s0}(t), \tag{17}$$

$$d(\Delta\Omega_{gis}) = \frac{dt}{J_i} \Delta M_{is}(t).$$

The procedure of integration of the equations of motion and calculation of the work is reduced to the summation of the differentials of variables and elementary work (products of unbalances of moments and the differentials of displacements).

The observer of the system motion, being in the situation that has developed by the time  $t_0$ , is not able to determine what margin of change in the potential energy of the system remains until the loss of stability. The reason for this is that the calculation of the work requires knowledge of the system trajectory, i.e. the integration of the equations of motion at  $t > t_0$ . However, some estimation of this margin can be made on the basis of the assumption about the smallness of the change in the object velocities (i.e. object velocities can be considered constant). Fixation of the object velocities at the time  $t_0$  allows us to predict the change in system angles in the simplest way, i.e. to determine their differentials without integration of the equations of motion [19].

In fact, this procedure is reduced to stopping the integration process at time  $t_0$  and calculating the work under the predicted uniform motion of objects (i.e. along the trajectory that continues the actual trajectory after the time instant  $t_0$ ). It is clear, that to correctly determine the moments acting in the system at each point of such a trajectory, the balance equations of active and reactive power at the nodes of the system must be satisfied. Such trajectories, satisfying only the laws of conservation of momentum of relative motions near the centers of inertia (3) and the equations of power balance, will be called possible [19, 23, 24]. On a possible trajectory, the equation of motion may not be satisfied. It appears that the invariability of the objects velocities along a possible trajectory is not necessary. It is enough to ensure their proportionality on this trajectory

(not constant speed of the objects, but the relationship between them).

The appearance of the extremum of the function that determines the dependence of the potential energy change in the displacement of the object in question  $\Delta W_{ob}(\Delta\delta_{ob})$  gives an estimate of its limit value and critical angle. If the kinetic energy of the object at time  $t_0$  exceeds the estimated margin of the potential energy variation, then we can expect stability loss at an angular displacement close to the critical one, in a time interval [8, 21-24]

$$\Delta T = \int_{\Delta\delta_{ob}(t_0)}^{\Delta\delta_{crit}} \frac{d\Delta\delta_{ob}}{\sqrt{\frac{2}{J_{ob}} \sqrt{K_{ob}(t_0) - \Delta W_{ob}(\Delta\delta_{ob})}}}, \tag{18}$$

where  $K_{ob}(t_0)$  – kinetic energy of the object at the time  $t_0$ ,  $\Delta W_{ob}(\Delta\delta_{ob})$  – change in the potential energy of the object as a function of its deviation from its position at time  $t_0$ . If  $K_{ob}(t_0)$  is selected equal to the change in potential energy that occurs from the initial position to its extreme value, at which  $K_{ob}(t) = 0$ , the motion on the interval  $(t_0 - t)$  will be along the limiting trajectory, where the stopping point coincides with the time when the potential energy extremum is reached. These estimates can be used to make control decisions to ensure stability at time  $t_0$ .

These considerations can form the basis of the algorithms designed to estimate the parameters of limit shock disturbances. A momentum disturbance is characterized by the values of velocity deviations at the time of disturbance elimination  $\Delta t$ , which determine the distribution of changes in the momentum and kinetic energy of objects in the system.

The distribution of momentum across the system (i.e. the relationships between the momentums of different objects) depends mainly on the place of application of the disturbance. If we focus on a specific distribution of the momentum at some severity of the test disturbance that does not lead to a stability loss, the increase in its severity can be modeled by a proportional increase in the amplitude of the momentums of objects due to an increase in the duration of the emergency. The severity of the test disturbance is characterized by the kinetic energy of oscillations, which is acquired by the system during the interval of the momentum action

$$K_{first}^{test}(\Delta t) = K_{reg}^{test}(\Delta t) + \sum_s K_{loc}^{test}(\Delta t). \tag{19}$$

Let us consider a possible trajectory of motion that begins at the moment  $t_{+0}$  (characterized by the momentums of objects at this time) and continues until the stability loss. The object, whose potential function of displacement reaches the extremum first, is the culprit of the stability loss. The extreme value of the potential function of an unstable object shows the value of the maximum kinetic energy of oscillations to be transferred to this object by the disturbance

at the initial time to cause its instability. Knowing the kinetic energy of the object that leads to stability loss, we can determine the corresponding momentum of this object. The ratio between the object momentum limiting in terms of stability at the initial time and the object momentum calculated with a test disturbance  $\beta_{ob}^{\lim}$

$$\beta_{ob}^{\lim} = \frac{J_{ob} \Delta \Omega_{ob}^{\lim}}{J_{ob} \Delta \Omega_{ob}^{test}} = \frac{\Delta \Omega_{ob}^{\lim}}{\Delta \Omega_{ob}^{test}}, \quad (20)$$

shows the relationship in which the momentums of all objects should change when the severity of the test disturbance changes to the level necessary for the loss of stability of the object in question. Then the value of the limit disturbance (the maximum kinetic energy of oscillations) can be determined based on the information about the test disturbance

$$K_{first}^{\lim}(\Delta t) = (\beta_{ob}^{\lim})^2 K_{reg}^{test}(\Delta t) + (\beta_{ob}^{\lim})^2 \sum_s K_{loc}^{test}(\Delta t). \quad (21)$$

We will first dwell on the estimation of maximum momentum disturbances in the first cycle of swings. This means that the consideration is given to the processes of the first quarter of the oscillation period, during which the maximum angular displacement of objects relative to the initial position occurs. The result of the momentum action is the formation of a momentum distributed across the system  $J_i \Delta \Omega_{gi} = \Delta M_i \Delta t$  by the time of the restoring switching  $\Delta t$ . With the known distribution  $J_i \Delta \Omega_{zi}$ , we determine  $\Delta \Omega_o(\Delta t)$  – variation in the velocity of the center of inertia of the system and the magnitude  $\Delta \Omega_{i0}(\Delta t) = \Delta \Omega_{zi} - \Delta \Omega_o(\Delta t)$  – deviations of the machine rotational velocities from the center of the inertia of the system. These deviations determine free motions in translational ( $\Delta \Omega_o(\Delta t)$ ) and in oscillatory ( $\Delta \Omega_{i0}(\Delta t)$ ) degrees of freedom of the system, respectively.

Since only the relationships between the momentums are important for the calculations, the magnitude  $\Delta t$ , unknown in advance, can take any value (when it changes, the relationships between the momentums of objects do not change). In practical calculations, it is convenient to choose it equal to unit. Emphasizing this circumstance, we denote this value as  $\Delta t_{calc}$ .

Let us choose one, for example, the  $k$ -th oscillatory degree of freedom and corresponding oscillator. Oscillatory impulses  $J_i \Delta \Omega_{i0}$  split between its synchronous and local motions. This means that each of the subsystems of the corresponding oscillator acquires a certain impulse with its sign and amplitude. Since we consider a certain disturbance, in the general case, the adjacent subsystems acquire the momentums that are not necessarily of an opposite sign. The oscillatory structure of motion for this emergency, characterized by antiphase motions of the adjacent subsystems, is obtained from the oscillating structure after

the merger of its adjacent subsystems with the same momentum signs. The kinetic energy of the oscillatory motion under this disturbance will be divided into regional and local components

$$\Delta K_{osc}(\Delta t_{calc}) = \Delta K_{reg}(\Delta t_{calc}) + \sum_s \Delta K_{locs}(\Delta t_{calc}), \quad (22)$$

what is characterized by the ratio  $N_k$ :

$$N_k = \frac{\Delta K_{reg}(\Delta t_{calc})}{\Delta K_{osc}(\Delta t_{calc})},$$

that determines the proportion of regional (synchronous) motions of the considered structure in the kinetic energy of oscillations. The distribution of the oscillation energy between synchronous machines of the system can be considered in terms of various oscillators of the system, in each of which the weight coefficient of the energy of synchronous motions is determined. We can assume that the oscillators in which the weight of synchronous motions has the maximum value, will belong to the dominant part of the excited oscillators. In this case, it means that these oscillators maximally resonate with the disturbance distributed in the form of a momentum.

In the event that the assessment reveals an instability structure, the value  $\Delta t_{emerg}$  of the duration of the considered emergency leading to a loss of stability is determined with the revealed structure of motion. The relationship  $\beta = \frac{\Delta t_{emerg}}{\Delta t_{calc}}$  shows in what ratio the momentums of the

objects of the oscillatory structure must change at the initial time in order to cause a loss of stability with this structure of unstable motion. This allows us to estimate the maximum kinetic energy of oscillations in terms of stability  $\Delta K_{osc}^{\lim}$ , which the disturbance should give to the oscillatory degrees of freedom of the system for the stability loss to occur with the revealed structure of motion

$$\Delta K_{osc}^{\lim}(\Delta t_{emerg}) = \beta^2 \left[ \Delta K_{reg}(\Delta t_{calc}) + \sum_s \Delta K_{locs}(\Delta t_{calc}) \right]. \quad (23)$$

In the described algorithm the emergency is represented as a short-term impulse action, during which the moment imbalances remain unchanged.

The assumption about the constancy of the moment imbalances makes it possible to approximately determine the kinetic energy of the oscillations  $\Delta K_{osc}(\Delta t)$ , input to the system in case of an emergency (independent of the system structure) as a quadratic function of its duration  $\Delta t$

$$\Delta K_{osc}(\Delta t) = 0.5 \sum_i J_i \Delta \Omega_{i0}^2(\Delta t) = 0.5 \left( \sum_i \frac{\Delta M_i^2}{J_i} - \frac{\Delta M_E^2}{J_E} \right) \Delta t^2. \quad (24)$$

The coefficient at the squared time interval  $\Delta t^2$  in (24) characterizes the susceptibility of the oscillatory degrees of freedom of the system to the emergency and determines the processes of kinetic energy accumulation in them.

An important characteristic of an unstable motion is the time necessary to achieve critical displacements after elimination of limit disturbance in terms of stability. This value can be determined with the help of relationship (18), in which the kinetic energy is equal to a certain maximum kinetic energy of the subsystems of the instability structure, and the potential energy is equal to the braking work (taken with an opposite sign) performed on the way of these subsystems after elimination of the emergency before they reach the limiting displacements. In addition, we can similarly calculate the time to reach the limiting displacement for the disturbances beyond limits.

Having carried out similar calculations for other oscillators of the system, we obtain an estimate of the spectrum of limit disturbances in the considered place of the system that lead to stability loss in various cutsets  $s_i$  of the power system in oscillators named « $f_i$ ». It is convenient to display the calculation results for multiple oscillators on an energy-time diagram. An example of such a diagram is shown in Figure 2 for three dominant oscillations of a certain power system (115 nodes, 33 synchronous machines) at a short circuit in a selected node.

The spectrum of "small" self-oscillations of this system contains 32 components. The oscillations of the first ten of them have non-star-like structures. The horizontal line segments show the energy levels of the kinetic energy of the oscillations, which the system must have after switching off the short circuit for the development of instability of various oscillators (energy levels of excitation of the instability structures of oscillators). The length of these segments shows the time that will pass after the short circuit is switched off until the moment of stability loss under the limit disturbance. With the disturbances beyond the limit, the time to reach the critical point decreases (energy-time "tails" of the beyond-limit disturbances that are directed upwards and to the left appear in each horizontal line). The cutsets of instability structures are given in the form of numbers of subsystems of oscillating structures between which this cutset is located. The parabola going from zero shows the dependence of the

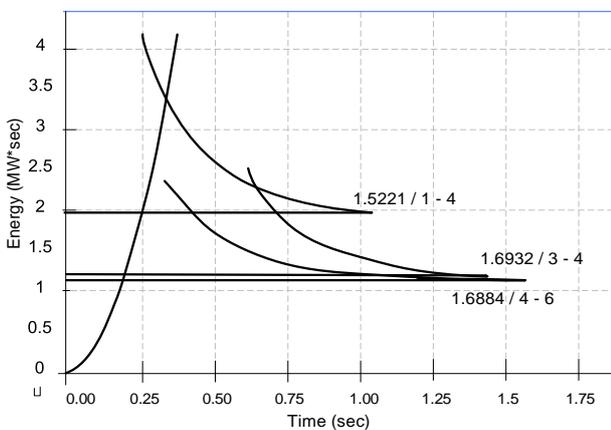


Figure 2. Time-energy diagram of instability.

acquired kinetic energy of oscillations on the duration of the emergency.

An analysis of the cutsets of the three obtained instability structures in the electrical network of the system shows that for the considered emergency all of them coincide (this is the same cross section). As is seen from the Figure, with a short circuit lasting less than ~0.2 seconds, the stability loss will not occur. With an increase in the short circuit duration above 0.2 sec, the stability loss associated with the oscillations of the oscillating structure of 1,6884 Hz, will manifest itself in the first place. The Figure also demonstrates that with a short-circuit duration of ~0.2 seconds, the stability will be lost in ~1.5 seconds after the short-circuit clearing, i.e. there is a time resource for emergency control to maintain stability, equal to 1.5 seconds. Accordingly, with the growth of the short circuit duration, this time resource is reduced. Testing the obtained results by direct calculation of the transient process shows their good match, both in terms of the short-circuit duration limit, and the location of the out-of-step cutset.

If we assume that the oscillatory structures of the spatial oscillators in the second cycle of oscillations coincide with the oscillatory structures of the first cycle, and the momentums of the subsystems only change sign, then it is also possible to estimate the stability loss conditions in the second cycle. The algorithm for this calculation does not differ much. The differences are only in calculating the time when the subsystems of the instability structure reach the limiting displacements, since in this case it is necessary to take into account the time spent on the first cycle.

Figure 3 shows the energy-time diagram of instability in the first and second swing cycles, calculated at a short circuit at another node of the same power system. Two dominant oscillating structures were considered, and it appeared for one of them (1.5521 Hz) that the stability could be lost in the second cycle. The corresponding curve is shown as a dotted line. It is seen that the instability in the second cycle of the cutset 1 – 4 of this structure develops over a long period of time.

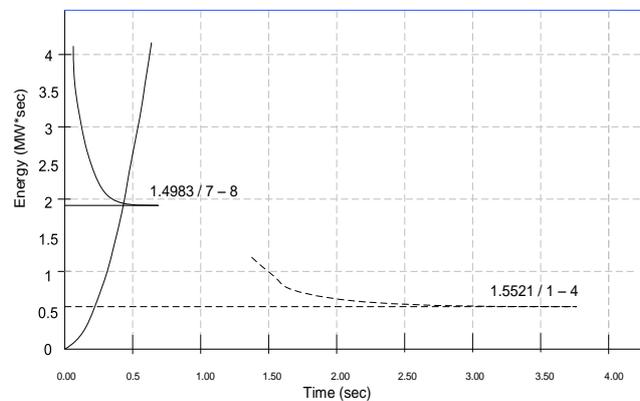


Figure 3. Time-energy diagram of instability in the first and second swing cycles.

With a larger magnitude of disturbance sufficient for the instability development in the first cycle, the instability will be observed in the form of a rapid development of the out-of-step conditions in the cutset 7 – 8 of the spatial oscillator 1,4983 Hz that does not coincide in the network with the previous cutset.

The instability characteristics shown in Figure 3 can be confirmed by the calculation of transient processes in a sufficiently complete simulation of the power system (taking into account speed controllers, power system stabilizers and excitation systems, damping rotor circuits). In Figure 4, the results of calculation of the transient process in the power system is represented by hodographs of voltage vectors at different nodes of the system (only a fragment of its scheme is shown). The transient process is caused by the 0.2 s connection of a short circuit shunt at node 5220 (the short circuits at this node are represented by the energy-time diagram in Figure 3). The hodographs show that the out-of-step conditions in the system are developing in the cutset formed by lines 2200-4202 and 4201-4203. Since nodes 4202 and 4203 are located practically in the centers of oscillation, lines 4205-4202 and 4205-4203 can also be considered to be the out of step cutset (adjacent cutset). The state shown in the Figure is reached in 3.3 second after connecting the short-circuit shunt.

Figure 5 demonstrates the calculation results for the transient process in the same scheme (the initial conditions) in case of a short circuit at the same node when it lasts 0.35 s. In the system, the out-of-step conditions are developing in the cutset constructed from the lines 5128-5122 and 5128-5126 (or 5122-5124 and 5126-5124). The state fixed in the Figure occurred in 1.1 s after connection of the short- circuit shunt.

In both examples, the voltage vectors in the out of step part of the system rotated a little more than one complete revolution with respect to the reference axis of the angles (which is represented by the longitudinal axis of rotor of the most powerful synchronous machine in the system). Considering the motion of the voltage vectors to the loss of stability, it can be established that in the first case this disturbance occurs in the second cycle of swings, and in the second – in the first cycle. With an intermediate value of the short circuit duration (between 0.2 and 0.35 sec.), the out-of-step cutset does not change its position and coincides with the first of the above cases. When the short circuit lasts less than 0.2 s no loss of stability is observed.

Out-of-step cutsets obtained by constructing energy-time diagrams coincide with one of the cutsets identified in the calculation of the transient process in the considered emergencies. The situation presented in Figure 3 determines the change in the position of the out-of-step cutset with an increase in the severity of the disturbance at some place of its location. It refers to the cases in which this change is associated with instability at different swing cycles. Such effects, however, can sometimes be observed

within one swing cycle (usually, not the first). In this case, a more energy-intensive instability case outpaces the development of a less energy-intensive one, and the out-of-step cutset is usually situated closer to the emergency site.

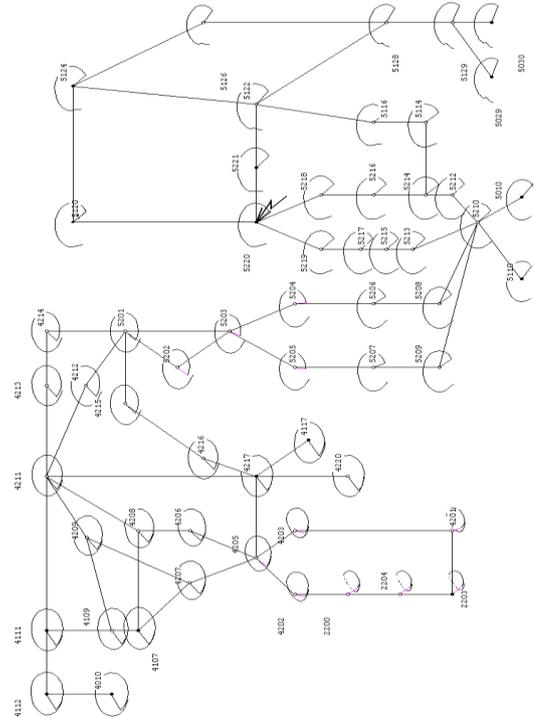


Figure 4. Stability loss in the system under a short circuit at node 5220 with a duration of 0,2 second.

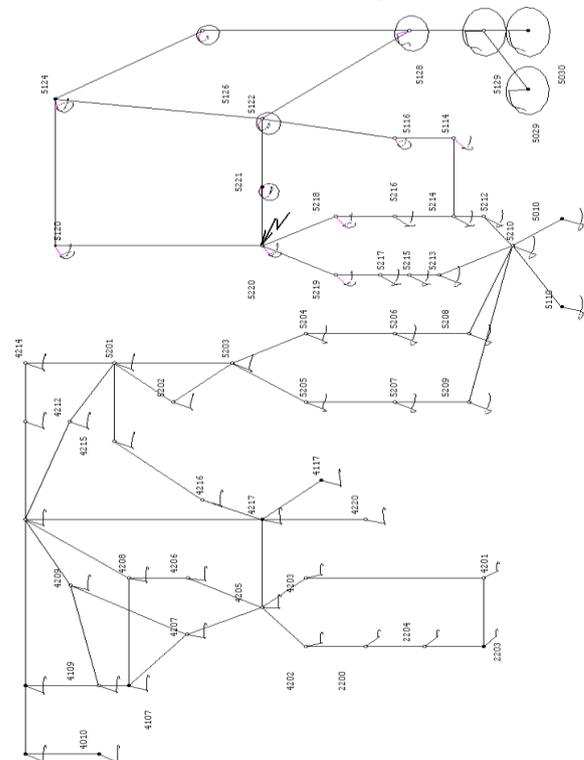


Figure 5. Stability loss in the system under a short circuit at node 5220 with a duration of 0,35 second.

The described algorithm for estimation of limiting impulse disturbances is, in fact, an extension of the known equal areas method to the oscillatory structures, which can contain more than two subsystems. The use of a possible trajectory of motion allows us to calculate the kinetic energy stored in the stage of emergency condition in the objects of the oscillatory structure suitable for describing the motion after the emergency elimination (after restoring switching), based on the calculation of the work on the motion of these objects in emergency conditions. In this case, for the impulsive transients, the oscillatory structure of the post-emergency condition is based on the oscillatory structures of the pre-emergency scheme. The damping of the acquired kinetic energy is determined by the calculation of braking work of these objects in the post-emergency conditions [23, 24]. These publications describe in more detail the method of calculations and their results for a particular system, up to the construction of power characteristics in the power-angle coordinates for the cutset with developing out-of-step conditions.

#### IV. THE RESEARCH ON THE PROCESS OF SYNCHRONISM LOSS IN A POWER SYSTEM BASED ON INTEGRATION OF ITS EQUATIONS OF MOTION

In addition to survey studies on stability, it is necessary to analyze the conditions of development and characteristics of unstable motion in terms of specific network structure, operating conditions, and emergency situations. This can be done in detail in the simulation of electromechanical transients in the power system based on the calculation of its equations of motion [25]. The resulting trajectories of the system satisfy the equations of the mathematical model and they can be attributed, on this basis, to the actual trajectories of motion. In this case, it is assumed that the mathematical model is adequate to the described object.

Loss of stability is linked to the attainment of critical conditions under the mutual motion of objects in an inhomogeneous system. The detection of the oscillatory structure changing in time in the transient process is based on the basic topological property of wave motion, namely, that its crests and troughs coexist in space. The boundaries between the crests and troughs can most likely be included in the out-of-step cutsets where loss of stability and loss of synchronism occur. Subsystems of oscillatory structure represent positive and negative half-waves. In general, these half-waves can represent the spatial distribution of accelerations, velocities, or displacements.

The structural organization of motion is based on the formation of groups of synchronous machines moving relative to the center of inertia of the system in a similar way. Such groups (the cores of subsystems) form the areas of the system around themselves, and the voltage vectors at the nodes of these areas adopt the nature of the group motion [25]. Let us choose, for example, a sign of deviation

of their rotation speed from the rotation speed of the center of inertia of the system  $\Delta\Omega_{gi0}(t)$  as a group feature for synchronous machines lying inside the selected area. This means that the sign of all variables

$$\Delta\Omega_{gi0}(t) = \Omega_{gi}(t) - \Omega_0(t) \quad (25)$$

at time  $t$  must be the same for all machines in the group forming the core of the subsystem. The system nodes that fall under the influence of the group will be determined on the basis of a similar requirement for the signs of deviations

$\Delta\Omega_{i0}(t)$  of their individual frequencies  $\Omega_i(t) = \frac{d\theta_i(t)}{dt}$  from the rotational speed of the center of inertia of the system  $\Omega_0(t)$  (the relative individual frequencies)

$$\Delta\Omega_{i0}(t) = \frac{d\theta_i(t)}{dt} - \Omega_0(t), \quad (26)$$

where  $\theta_i(t)$  the angle of the voltage vector at the node relative to the fixed axis. When calculating electromechanical transients, the angles of the voltage vectors and the rotors of synchronous machines are measured relative to a somehow selected common reference axis, usually a rotating one. The time derivatives for the angles measured in such a way are calculated using the following relations:

$$\frac{d\delta_i(t)}{dt} = \Omega_i(t) - \Omega_{ref}(t) \quad (27)$$

where  $\Omega_{ref}(t)$  - the angular velocity of the reference axis,  $\delta_i(t)$  - angle relative to it. Then, with this measurement of angles, we obtain:

$$\Delta\Omega_{i0}(t) = \Omega_i(t) - \Omega_0(t) = \frac{d\delta_i(t)}{dt} + \Omega_{ref}(t) - \Omega_0(t). \quad (28)$$

One subsystem will include all the nodes of the system, at which the individual frequencies are either higher or lower than the angular velocity of the center of inertia of the system.

To identify subsystems, we determine the boundaries between them. In this case, the attribute of the boundary tie connecting two adjacent subsystems becomes a different sign of deviations of the individual frequency at the nodes at its ends. If we assume all such ties of the system at time  $t$  to be disabled and make a topological analysis of the system to identify disconnected subsystems, the common feature for the resulting subsystems will be the same sign of deviations of individual frequencies within the subsystem, and any adjacent subsystems will be characterized by different signs of these deviations.

The described algorithm allows determining the oscillatory structure of motion in the system as a function of time  $S(t)$ . This oscillatory structure defines the spatial distribution of the relative velocities of the system as an

electromechanical wave, and highlights alternating areas that outrun or fall behind the center of inertia of the system.

Relative displacements can also be used to determine the structure of motion. Since the stability of electromechanical oscillations is determined by the work carried out on the trajectory of motion, the studies usually consider the displacement wave, which determines the work done by redundant moments in the transient process [25].

Displacement of the voltage vector of the node relative to the center of inertia of the system, accumulated on the interval  $(t_0-t)$   $\Delta\delta_{i0}(t, t_0)$  given (28) will be determined as follows:

$$\Delta\delta_{i0}(t, t_0) = \int_{t_0}^t \Delta\Omega_{i0} dt = \delta_i(t) - \delta_i(t_0) - \int_{t_0}^t (\Omega_0 - \Omega_{ref}) dt. \quad (29)$$

The calculation of the integral on the right-hand side gives:

$$\Delta\delta_{i0}(t, t_0) = \delta_i(t) - \delta_i(t_0) - \frac{\sum_k J_k [\delta_{gk}(t) - \delta_{gk}(t_0)]}{J_E}, \quad (30)$$

where  $\delta_{gk}(t) - \delta_{gk}(t_0)$  - change in the angle of the  $k$ -th synchronous machine on the interval  $(t_0-t)$ , and the summation is performed for all synchronous machines of the system. The angles of voltage vectors and angles of rotors of synchronous machines measured relative to the reference axis used in the calculation are used in the right-hand part of (30).

After  $\Delta\delta_{i0}$  is calculated for all nodes of the system, and the ties connecting the nodes with different signs of these deviations are marked, we can similarly identify the oscillatory structure of the system motion. Here, we employ the relative displacements accumulated over a finite time interval, and represent the result of the motion of the system distributed in the nodal space by an electromechanical displacement wave. The alternating areas of the system are shifted with respect to the vector rigidly connected with the center of inertia of the system to the positive or negative sides (half-waves of integral relative displacements).

Since the oscillatory structures are associated with the motion of large masses, they evolve rather slowly and it is possible to determine the time intervals during which the oscillatory structure is unchanged. The process of the oscillatory structure evolution in time and space is associated with an unsteady wave process in a significantly inhomogeneous system. Of greatest interest are the structures that form in the system by the time when the development of the out-of-step conditions starts. It is these structures that must be considered to determine emergency control.

The relations for the determination of kinetic energy of regional motion and its total time derivative for the variable oscillatory structure have the form:

$$K_{reg}(S(t)) = \sum_{s(t)} J_s \frac{\Delta\Omega_{s0}^2(t)}{2}, \quad (31)$$

$$\frac{dK_{reg}}{dt}(S(t)) = \sum_{s(t)} \Delta M_{s0}(t) \Delta\Omega_{s0}(t). \quad (32)$$

It can be seen that the determination of the regional characteristics of the transient process in a variable oscillatory structure is reduced to a simple recalculation of the relative velocities and moments for the newly identified subsystems. These relations can be interpreted as a way of understanding the speed, power and energy characteristics of the transient process over its entire time period in the context of the structure  $S(t)$  that has developed by a certain time  $t$ .

The kinetic energy of an object on the interval  $(t_0-t)$  may increase or decrease. In the first case, the work is positive, in the second - negative. Since work  $A$  can be linked to a change in potential energy  $\Delta U$  (according to the definition of the latter as  $\Delta U = -A$ ), then the positive work corresponds to a descent into a potential well. Negative work corresponds to an ascent from the potential well. It is worthwhile to note that the idea of a potential well due to the presence of non-potential forces in the power system is not strict, but it makes it possible to increase the visibility of the physical picture of oscillations and instability. The study of the structural organization of oscillations, that involves the identification of regional objects (subsystems), allows determining the spatial and temporal characteristics of transmutation of kinetic and potential energy of the system in the transient process.

The physical cause of the instability in the transient process is the insufficient resources for braking divergent objects (subsystems). In the case of structurally organized motion, this means that the kinetic energy of regional motion is too large and the synchronizing torques are unable to stop the diverging subsystems. Using the idea of an object moving in a potential well, we can say that the instability is associated with the fact that this object has attained the maximum of a potential function, behind which there is another potential well, with a different equilibrium position (if it exists) [25].

Let us consider pairs of coupled subsystems of the oscillatory structure. These pairs can contain both subsystems of oscillatory structure and subsystems formed as a result of complete or partial merging of subsystems located on different sides of the considered cutset. Thus, the pair can cover part of the system or the whole system. For simplicity, we assume that the subsystems included in it have numbers 1 and 2.

Determine a relative velocity of the center of inertia of the subsystem that *contains both subsystems* of the pair relative to the center of inertia of the system  $\Delta\Omega_{(1+2)0}$  on the basis of the relation:

$$J_1 \Delta\Omega_{10} + J_2 \Delta\Omega_{20} = (J_1 + J_2) \Delta\Omega_{(1+2)0},$$

where  $\Delta\Omega_{10}(t)$  and  $\Delta\Omega_{20}(t)$  are the deviations of the velocities of the first and second subsystems of the pair

relative to the center of inertia of the system;  $J_1$  and  $J_2$  are the moments of inertia of the subsystems. Deviations of the velocities of the subsystems from the velocity of the center of inertia of the pair  $\Delta\Omega_{1(1+2)}$  and  $\Delta\Omega_{2(1+2)}$  are equal to:

$$\begin{aligned} \Delta\Omega_{1(1+2)} &= \Delta\Omega_{10} - \Delta\Omega_{(1+2)0}, \\ \Delta\Omega_{2(1+2)} &= \Delta\Omega_{20} - \Delta\Omega_{(1+2)0}. \end{aligned} \quad (33)$$

For these velocity deviations, the ratio will always be satisfied:

$$J_1\Delta\Omega_{1(1+2)} + J_2\Delta\Omega_{2(1+2)} = 0. \quad (34)$$

Two equations of motion of the subsystems of the pair relative to its own center of inertia will have the form:

$$J_1 \frac{d(\Delta\Omega_{1(1+2)})}{dt} = \frac{\Delta M_1 J_2 - \Delta M_2 J_1}{J_1 + J_2} = \Delta M_{1(1+2)}, \quad (35)$$

$$J_2 \frac{d(\Delta\Omega_{2(1+2)})}{dt} = \frac{\Delta M_2 J_1 - \Delta M_1 J_2}{J_1 + J_2} = \Delta M_{2(1+2)}, \quad (36)$$

where  $\Delta M_{1(1+2)}$  and  $\Delta M_{2(1+2)}$  - the relative excess moments of subsystems 1 and 2 determining their motion near the center of inertia of the pair, and  $\Delta M_{2(1+2)} = -\Delta M_{1(1+2)}$ .

The kinetic energy of the regional oscillations  $K_{reg(1+2)}$  of the pair is determined as a sum of the kinetic energy of the two subsystems:

$$\begin{aligned} K_{reg(1+2)} &= 0.5J_1\Delta\Omega_{10}^2 + 0.5J_2\Delta\Omega_{20}^2 = \\ &= 0.5(J_1 + J_2)\Delta\Omega_{(1+2)0}^2 + 0.5J_1\Delta\Omega_{1(1+2)}^2 + 0.5J_2\Delta\Omega_{2(1+2)}^2 \end{aligned} \quad (37)$$

As is seen from (37), part of the kinetic energy is associated with the general motion of the pair, and there is a component determined by the internal regional oscillations of the subsystems of the pair near its center of inertia  $K_{int(1+2)}$

$$K_{int(1+2)} = 0.5J_1\Delta\Omega_{1(1+2)}^2 + 0.5J_2\Delta\Omega_{2(1+2)}^2 \quad (38)$$

It follows from (35) and (36) that the change  $K_{int(1+2)}$  is equal to the integral of work of the relative moment on the mutual displacement of subsystems  $\Delta\delta_{12}$ :

$$\int_{t_0}^t dK_{int(1+2)} = \int_{\Delta\delta_{12}(t_0)}^{\Delta\delta_{12}(t)} \Delta M_{1(1+2)} d(\Delta\delta_{12}), \quad (39)$$

where  $d(\Delta\delta_{12}) = \Delta\Omega_{12} dt$  and

$$\Delta\Omega_{12} = \Delta\Omega_{1(1+2)} - \Delta\Omega_{2(1+2)} = \Delta\Omega_{10} - \Delta\Omega_{20}.$$

The known method of transformation of differential equations of motion applied to the pair leads to a similar relation. The first of them is multiplied by  $J_2$ , the second - by  $J_1$ , then, the second is subtracted from the first one. As a result, we obtain one differential equation of mutual motion of subsystems:

$$J_{red} \frac{d(\Delta\Omega_{12})}{dt} = \Delta M_{1(1+2)} \quad (40)$$

The excess moment in the right-hand side of (40), which determines the mutual motion of the subsystems, coincides with the excess moment acting on the first subsystem in its relative motion near the pair's center of inertia. The value  $J_{red} = \frac{J_1 J_2}{J_1 + J_2}$  is the reduced moment of inertia. Kinetic energy of mutual motion is described by equation (40):  $0.5J_{red}\Delta\Omega_{12}^2$ .

If (40) is converted into an integral relation of type (39), it can be established that the kinetic energy of mutual motion of the subsystems of the pair is equal to the total kinetic energy of oscillations of the subsystems of the pair relative to its own center of inertia [25].

The possibility of calculating the work for pairs of adjacent subsystems of the oscillatory structure depending on their relative displacement  $\Delta\delta_{12}$  (i. e. on one coordinate), allows a schematic presentation of the potential well bounded on both sides by potential barriers that determine a margin for negative work on the ascending sections of the trajectories of the first and second cycle (Figure 6).

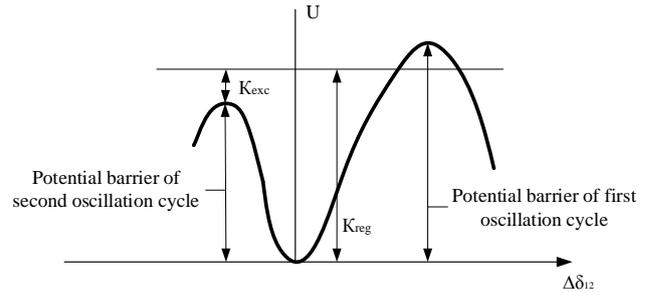


Figure 6. Determination of the excess kinetic energy leading to the loss of synchronism between the subsystems of instability structure in the second oscillation cycle.

If in this Figure we display the kinetic energy acquired by the subsystems, its comparison with the heights of the potential barriers will allow us to judge about the loss of synchronism in the system in the first or second oscillatory cycles (Figure 6 shows a case of instability in the second oscillatory cycle). The value of the kinetic energy, that was not converted into potential energy (in negative work),  $K_{exc}$  acts as excess kinetic energy, which is the cause of the instability.

The identification of the loss of dynamic stability with subsequent development of out-of step conditions and location of the out-of-step cutset should be performed automatically. Algorithms for such an analysis should be built into the program for calculation of transient processes. Note that these algorithms can be applied to any mathematical model used for numerical analysis of electromechanical transients. These algorithms should determine the main characteristics of the loss of synchronism in a power system for a given disturbance (if

the loss of stability occurs). The characteristics of the loss process can be: the time after which the out-of-step conditions start to develop in the system, the out-of-step cutset (or the cutsets under a multi-frequency out-of-step conditions), the excess kinetic energy that caused the development of the out-of-step conditions in this cutset. If stability is maintained, the algorithm should allow this to be established automatically.

Identification of a subsequent loss of stability based on an analysis of the extreme values of work done during the motion of an object is very difficult algorithmically. Therefore, the fact of a loss of stability is easier to establish based on the calculation of mutual displacements of the subsystems of a variable oscillatory structure. In the event that the mutual displacement of subsystems in the transient process reaches a certain limiting value, the development of an unstable motion is identified.

To identify the instability and determine an out-of-step cutset, it is necessary to identify the inter-system tie (inter-system ties) of the oscillatory structure, along which the out-of-step conditions develop. Such a tie can be determined by the calculations of changes in mutual angles between adjacent subsystems of the oscillatory structure. For the adjacent  $s$ -th and  $m$ -th subsystems, the change in the mutual angle  $\Delta\delta_{sm}(t, t_0)$  on interval  $(t_0-t)$  is determined as follows:

$$\Delta\delta_{sm}(t, t_0) = \Delta\delta_{s0}(t, t_0) - \Delta\delta_{m0}(t, t_0) = \frac{\sum_{k_s} J_k [\delta_{gk}(t) - \delta_{gk}(t_0)]}{J_s} - \frac{\sum_{k_m} J_k [\delta_{gk}(t) - \delta_{gk}(t_0)]}{J_m}, \quad (41)$$

where  $k_s$  and  $k_m$  - sets of generator nodes of the  $s$ -th and  $m$ -th subsystems. In this case, the intersystem tie in which the out-of-step conditions occur can be identified on the basis of:

$$|\Delta\delta_{sm}(t, t_0)| \geq 180^\circ. \quad (42)$$

If the disconnection of the detected intersystem tie leads to the division of the system into two separate parts, the determination of the out-of-step cutset is completed. Practical calculations show that the oscillatory structures with respect to displacements, which emerge in the system immediately before the out-of-step condition and at the initial stage of its development get extremely simplified. They often consist of two subsystems, and rarely of three and four subsystems. The cases of ring structures have not been observed in computational practice yet.

With instability between two adjacent subsystems of the oscillatory structure formed in the system, the development of the out-of-step conditions begins, and it is accompanied by an increase in their mutual angular displacement. We call such a pair of subsystems unstable [25]. Figure 7 shows an unstable pair. Depending on the number of subsystems of the oscillatory structure, the unstable pair can cover a part of the system or the whole system.

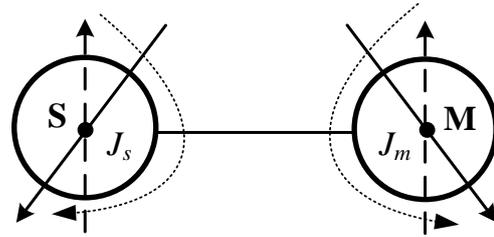


Figure 7. Unstable pair.

The oscillations of the subsystems of an unstable pair (with the numbers of subsystems 1 and 2) relative to its center of inertia are described by relationships that are similar to the analogous relationships for a simple two-machine scheme. The difference is that the excess moment determining the mutual motion of the subsystems depends on the total trajectories of the motion (regional plus local) of synchronous machines of the entire system (both outside and inside the unstable pair), and not only on the regional mutual displacement  $\Delta\delta_{12}(t, t_0)$ , as it would be for a real two-machine scheme [25].

A potential well can be constructed on the basis of the calculation of the transient process by plotting the kinetic energy of mutual oscillations at a certain time taken with a negative sign along the ordinate axis, and the mutual angle  $\Delta\delta_{12}(t, t_0)$  observed at the same time along the abscissa axis [25].

It is clear that the calculated potential wells reflect, in a different form, the same energy relationships that underlie the equal areas method, different from the latter by using actual trajectories. At the same time, the changes in potential energy on these trajectories are calculated not by calculating the work (which is difficult), but by determining the kinetic energy changes (which is much easier in algorithmic terms). The equal areas method is used to estimate the limit disturbances. The study of potential wells for the unstable pair is focused on the search for emergency control preventing the loss of stability.

Due to the presence of non-potential forces in the power system, the notion of a potential well is not strict. It is illustrative and in good agreement with everyday experience. The Figures below for the calculated potential wells are simplified. They, for example, do not show the discrepancy between the ascending and descending branches of the same swing cycle associated with damping, and the manifestations of local motion. Figure 8 shows the calculated potential well for the loss of synchronism in the first swing cycle. The area 1-2 is the downward slope of the potential well in the case of emergency conditions, 2-3 is the ascending branch of the first swing cycle in the post-emergency conditions. The sequence of positions in the potential well is 1-2-3.

Figure 9 shows the calculated potential well under loss of synchronism in the second oscillation cycle. The area 1-2 is a slope of the potential well in the case of emergency conditions, 3-2-4 - potential well in disaster mode.

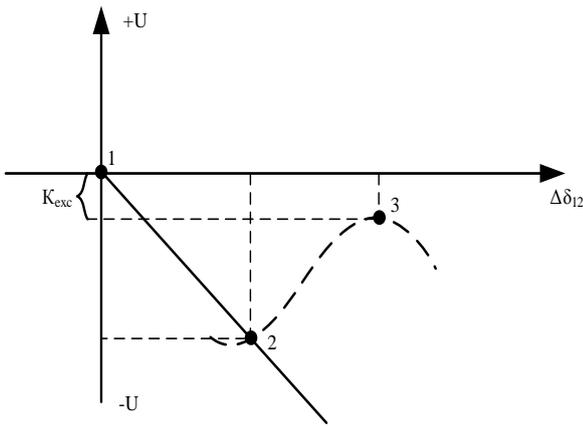


Figure 8. The calculated potential well under loss of synchronism in the unstable pair cutset in the first oscillation cycle.

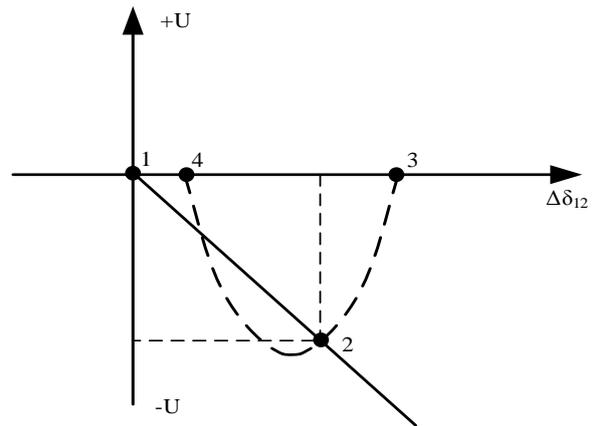


Figure 10. The calculated potential well of the pair of subsystems under steady motion in the first and second oscillation cycles.

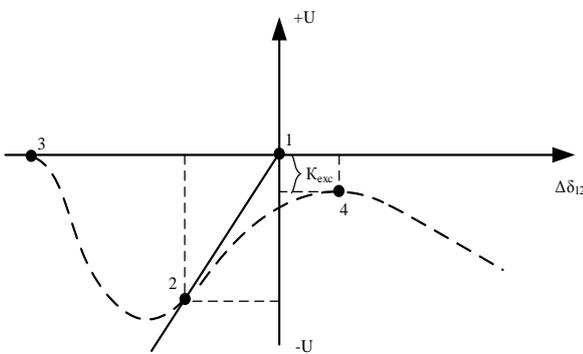


Figure 9. The calculated potential well under loss of synchronism in the unstable pair cutset in the second oscillation cycle.

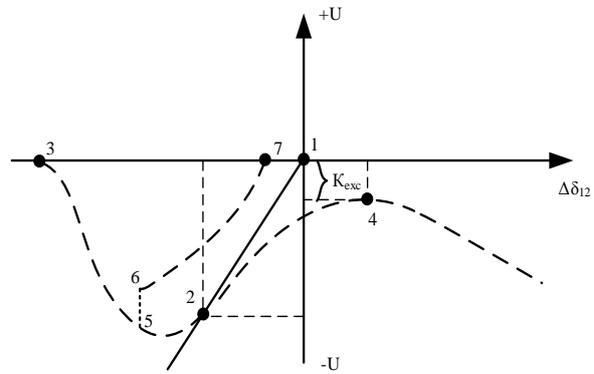


Fig. 11. Change in the calculated potential well under successful emergency control action preventing loss of synchronism in the second oscillation cycle.

Subsystem 1 running out at the loss of synchronism in the first cycle of swings is lagging behind. The sequence of positions is 1-2-3-2-4.

Figure 10 shows the steady mutual motion in the first and second swing cycles. The sequence of positions is: 1-2-3-2-4-2. These Figures are qualitatively identical for any pairs of adjacent subsystems of oscillatory structure (there is no unstable pair). If the stability is maintained in the subsequent cycles of swings, then, in the presence of damping, we can talk about reduction in the size of the potential wells of these pairs in the subsequent stages of motion in the vertical and horizontal directions (they are pulled into the point of stable equilibrium of the pairs in the post-emergency conditions). With negative damping (self-oscillation), the sizes of some potential wells increase, and their shape is distorted until an unstable pair appears.

At a fixed oscillatory structure (detected, for example, at the beginning of the out-of-step condition development), which is used to analyze the motion throughout the entire transient process, starting from a disturbance, and at the moments of switching that are not accompanied by a change in the inertial mass, the velocities of the inertia

centers of the system and subsystems, as well as their kinetic energy are unchanged. With changes in the rotating masses, these characteristics of motion also change.

The possible impact of emergency control on the type of the calculated potential well is shown in Figure 11. It demonstrates the case where the uncontrolled system falls out of synchronism in the second cycle of the oscillation shown in Figure 9. At the moment represented by position 5, located at the end of the descending branch of the first swing cycle, the generators are switched off. The change in kinetic energy between positions 5-6 is associated with a decrease in the inertial mass of the run-out subsystem. The area 6-7 represents an ascending stable branch of the second swing cycle. The sequence of positions inside the potential well in the first and second cycles of swings is 1-2-3-5-6-7-6.

## V. CONCLUSION

The wave nature of electromechanical oscillations determines the priority of the system motion structure in the formation of the transient process and its stability. In a complex system, it is the structure of the motion (what oscillates with respect to what), which is the main goal of

the study. It determines all the essential aspects of the process, starting with its specific features and stability and ending with the location of the out-of-step cutset. The wave approach to identifying the spatial structure of electromechanical oscillations allows us to study the processes of loss of synchronism in power systems under the influence of disturbances.

The study of these processes is based on the energy relations for structurally organized motion. The main method is to replace the problem of stability analysis in its classical formulation, which does not use the concept of "the structure of motion", with a set of small problems of stability analysis of structurally organized forms of the studied motion.

The description of structurally organized motion is based on the use of hierarchically built systems of coordinates. In the simplest case, we focus on the following: the translational motion of the center of inertia of the system, the oscillatory motion of the centers of inertia of subsystems relative to the center of inertia of the system, and synchronous machines relative to the centers of inertia of subsystems.

Using this hierarchical structure, we can analyze the processes of loss of synchronism in complex power systems on possible (without integration of equations of motion) and actual (with integration of equations of motion) trajectories by determining kinetic and potential energy of oscillations with identification of their regional and local components.

The loss of synchronism entails two physical processes:

- the process of formation and development of the oscillatory structure of motion as a consequence of the wave electromechanical process that determines the interactions within the system;
- the process of loss of stability of parallel operation (angle stability) between the arising adjacent subsystems of the oscillatory structure of motion developing in space and time.

The study of the processes of loss of synchronism is carried out on the basis of interrelated algorithms designed to determine:

- the energy characteristics of stability limit disturbances for survey studies using energy-time diagrams;
- the excess kinetic energy resulting in an unstable pair of subsystems for the analysis of stability in specific emergencies and the choice of emergency control;
- the time of the unstable motion development;
- the location of the out-of-step cutset.

#### REFERENCES

- [1] V.A. Venikov, *Transient electromechanical processes in power systems*. Moscow, Russia: Vysshaya shkola, 1978, 415 p. (in Russian)
- [2] P. Anderson, A. Fuad, *Power system control and stability*. John Wiley and Sons, Boston, 1975, 465 p.
- [3] E.S. Lukashov, *Introduction to the theory of electrical systems*. Novosibirsk, Russia: Nauka, 1981, 173 p. (in Russian)
- [4] E.I. Ushakov, *Static stability of electrical systems*. Novosibirsk, Russia: Nauka, 1988, 273 p. (in Russian)
- [5] A.A. Gorev, *Transient processes of a synchronous machine*. Leningrad, Russia: Gosenergoizdat, 1950, 386 p. (in Russian)
- [6] L.A. Zhukov, N.K. Krug, L.V. Yarnykh, "Quantitative estimation of equivalent admittance for transient stability calculations. Proc. of Academy of Sciences of the USSR," *Energy and Transport*, vol. 4, pp. 21-28, 1971. (in Russian)
- [7] K.M. Stanton, "Dynamic energy balance studies for simulation of power-frequency transient," *IEEE Trans. on Power Apparatus and Systems*, Vol. 91, No. 1, pp. 110-117, 1972.
- [8] L.D. Landau, E.M. Lifschich, *Mechanics (Theoretical physics)*. Moscow, Russia: Nauka, 1965, 280 p. (in Russian)
- [9] F. Krauford, *Waves*. Vol. 3. Moscow, Russia: Nauka, 1974, 527 p. (in Russian)
- [10] A.G. Phadke, J.S. Thorp, *Synchronized Phasor Measurements and Their Applications*, Springer Science + Business Media LLC, 2008, 248 p.
- [11] Yilu Liu, Shutang You, Ling Wu, Penn Markham, Dao Zhou, Yu Su, "Observation and Applications of Electromechanical Wave Propagation Based on Wide-Area Synchronous Measurement," *Preprints of the 20th World Congress The International Federation of Automatic Control*, Toulouse, France, July 9-14, 2017 pp. 75-80.
- [12] O.N. Agamalov, "Physical processes during damping of electromechanical oscillations of a synchronous machine with a modular-phase automatic excitation controller," *Electricity*, vol. 7, pp. 24-34, 2014 (in Russian)
- [13] A.A. Lysitsin, M.A. Edlin, "The algorithm for selecting control actions under the conditions of dynamic stability," *News of the NTC of the Unified energy system*, vol. 1 (68), pp. 41-47, 2013. (in Russian)
- [14] J. Machowski, J.W. Bialek, J.R. Bumby, *Power System Dynamics: Stability and Control*, 2nd Edition, John Wiley & Sons, Ltd., 2008, 629 p.
- [15] Y. Xu, F. Wen, G. Ledwich, Y. Xue, Electromechanical Wave in Power System: Theory and Applications, *Journal Modern Power System Clean Energy*, vol. 2(2), pp. 163-172, 2014.
- [16] L. Huang., M. Parashar, A. Phadke, J. Thorp, "Impact of electromechanical wave propagation on power system protection and control", *Cigre Session 2002*, Paris, France, 2002.
- [17] T. Li, G. Ledwich, Y. Mishra, J.H. Chow, A. Vahidnia, "Wave Aspect of Power System Transient Stability - Part 1: Finite Approximation," *IEEE Transactions On Power Systems*, vol. 32, no.4, pp. 2493-2500, 2017.

- [18] T. Li, G. Ledwich, Y. Mishra, J.H. Chow, A. Vahidnia, "Wave Aspect of Power System Transient Stability - Part II: Control Implications," *IEEE Transactions On Power Systems*, vol. 32, no.4, pp. 2501-2508, July 2017.
- [19] N.N. Lizalek, A.N. Ladnova, V.F. Tonyshev, M.V. Danilov, "Structure of electromechanical oscillations and stability of power systems," *Electricity*, vol. 6, pp. 11-19, 2011. (in Russian)
- [20] V.V. Bushuev, N.N. Lizalek, N.L. Novikov, *Dynamic properties of interconnected power*. Moscow, Russia: Energoatomizdat, 1995, 320 p. (in Russian)
- [21] N.N. Lizalek, *The dynamic properties of power systems with electromechanical oscillations. Structural organization of movements and sustainability*. Novosibirsk, Russia: NSTU, 2013. 212 p. (in Russian)
- [22] N.N. Lizalek, V.F. Tonyshev, *Forecasting and identification of instability in power systems*. Novosibirsk, Russia: NGAVT, 2013, 230 p. (in Russian)
- [23] N.N. Lizalek, A.N. Ladnova, V.F. Tonyshev, Assessment of dynamic stability of power systems based on the equal areas method, *News of the Russian Academy of Sciences, Energy*, vol. 1, pp. 139-152, 2013. (in Russian)
- [24] N.N. Lizalek, A.N. Ladnova, V.F. Tonyshev, E. Yu. Popova, A.S. Vtorushin, "Evaluation of the Transient Stability of Power Systems Based on the Area Method," *Thermal Engineering*, Vol. 60, No. 13, pp. 938-947, 2013.
- [25] N.N. Lizalek, V.V. Vasilev, D.N. Borodin, Arzhannikov S.G., "Selection of corrective control to ensure dynamic stability of the wave electromechanical process in the power system," *Electricity*, vol. 2, pp. 4-13, 2017. (in Russian)
- [26] M. Pavella, D. Ernst, D. Ruiz-Vega, *Transient stability of power systems. A unified approach to assessment and control*. Boston, Kluwer Academic Publishers, 2000, 237 p.



**Nikolay N. Lizalek** is currently a Professor of the Department of automated electric power systems of Novosibirsk State Technical University, Doctor of Technical Sciences, Professor. He graduated from the Department of Electric Power of Novosibirsk Electrotechnical Institute in 1969. In 1976, he defended his thesis «The study of electromagnetic transients of synchronous machine operating in a circuit with complex operator resistance», and in 1994 he defended his doctoral thesis «Analysis of the dynamic properties of power connections based on the wave approach».



**Vladimir V. Vasilev** graduated from Novosibirsk State Technical University with a degree in relay protection and automation of electric power systems in 2007. In 2011, he defended his thesis «Development of automation of complex emergency load shedding». Currently, he works as a Chief Expert in the Department of Emergency Control of Relay Protection and Automation Service in the Branch of JSC «System Operator of the Unified Power System» - «United dispatching control of the South power system» and is an associate Professor of the Department of Power Plants of the Novosibirsk State Technical University.