An Analysis of Shortage Minimization Models to Assess Power System Adequacy

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***Abstract* — Continuous changes and expansion of power systems lead to their complexity and aggregation. Therefore, the existing models and software to calculate the reliability of such systems can be inefficient from the standpoint of time, accuracy and adequacy of the results. To obtain objective information, we analyzed some existing power shortage minimization models that are applied to assess the adequacy of a power system. The paper presents the results of the model studies on the availability of a physically incorrect power flow between nodes and two-sided flows. The studies have proven the existence of a set of optimal solutions. The existing approaches considering adequate power flow have been tested. To cope with the described problems we have proposed additional constraints on power flows, and a two-stage method for power flow optimization, which, in the end, enabled the revealed problems to be resolved.**

***Index Terms* — adequacy, optimization methods, power shortage minimization, two-stage power flow model.**

I. Introduction

The electric power industry is a basic industry for successful development and operation of the economy and it must be compatible with the consumer requirements for reliable power supply. Reliability is an important complex property of power systems, which is understood as their capability to supply power to consumers in a required volume and with a required quality. In the stage of power system expansion planning and direct operation, the required reliability level should be assessed and the control actions and plans on commissioning of new system components and retirement of outdated ones should be corrected in due time.

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The current situation is that the expansion of power systems leads to their aggregation, and an increase in the number of generating components and transmission lines. These facts, in turn, dictate the requirements for development of computational tools to assess power system reliability, since the indicated trends decrease the computational efficiency (from the viewpoint of the time spent and validation of calculations) in the process of system reliability assessment because of "outdated" methods and algorithms used.

There are some models and software designed to assess power system adequacy: "MEXICO" model (EDF, France) [1, 2], "SICRET" model (ENEL, Italy) [1, 3], "COMREL" model (University of Saskatchewan, Canada) [4, 5], "POTOK-3" model (SEI SB RAS) [6]. However, these models are not used now. It should be noted that the subject-matter of the adequacy remains topical and is evolving. Therefore, such models and modules as GE "MARS" [7], GridView [8], MARELI [9], PLEXOS [10], ORION / ORION-M model (Komi Research Center) [11], YANTAR model (ESI SB RAS) [12] evolve, gain popularity and are applied to adequacy assessment of present-day power systems.

These products are used to determine the optimal reserve of the power generation and to choose a rational network structure for electric power systems. Therefore, the models represented by the linear minimization problem as well as the highly simplified problem statements distort the assessment results when determining the mathematical expectation of the electrical load undersupply to the system facilities, which, in turn, affects the level of reserve to be determined.

We also consider the "Nadezhnost" software that has been developed recently. This software is intended to study different optimization methods to estimate the efficiency and identify the methods capable of solving the problems with a great number of variables.

Note that the majority of programs apply the algorithm based on the Monte Carlo method [12]. This algorithm includes different mathematical models of optimal power flow with different statements and methods for solving the optimization problems [13]. The algorithm for assessment of power system adequacy is based on the Monte Carlo method and comprises three basic blocks:

1) A probability block for generation of power system

states with components randomly put in or removed from operation.

2) A block for power shortage calculation intended for power shortage minimization in each generated system state.

3) A block for calculation of power system reliability indices, which is designed to process and analyze all the stored information (the result of work of two previous blocks) and calculate such adequacy indices as the probability of failure-free (shortage-free) operation, the mathematical expectation of power shortage in power systems, the mathematical expectation of power undersupply, etc.

The core of the considered algorithm is the second block responsible for the calculation of power shortage in different power system states. The quality of the results, which implies the calculation speed and accuracy, the capability to solve the problems with an increasing number of variables, depends on the applied optimization method and the model correctness. In the end, the minimal difficulties or delays in calculations increase the time spent on solving the whole problem. Thus, the goal is to solve the power shortage minimization problem as fast as possible within a short space of time.

The research is mainly focused on an analysis of power shortage minimization model, which takes into consideration quadratic power losses, conformity of the model to real physical processes, application of different optimization approaches and methods, namely, a combination of the penalty function method and the gradient descent method.

II. Problem statement

The problem of power shortage minimization is formulated as follows:

Determine an optimal power flow in a power system for the known values of generating capacities in operation, the required levels of consumer loads, the transfer capabilities of tie lines in a power system and the factors of power losses in the system tie lines [12]. There exist several types of power shortage minimization models, and this paper deals with the models applied. The linear statement of the problem is presented below.

Mathematically, the problem is formulated as follows:

$$\sum_{i=1}^{n}(\grave{y}_i - y_i) \to \min_{y}, \tag{1}$$

subject to the balance constraints:

$$x_i - y_i + \sum_{j=1}^{n}(1-a_{ji})z_{ji} - \sum_{j=1}^{n} z_{ij} = 0, \quad i = 1,\dots,n, i \neq j, \tag{2}$$

and to the constraints on optimized variables:

$$0 \le y_i \le \grave{y}_i, i = 1,\dots,n, , \tag{3}$$

$$0 \le x_i \le \grave{x}_i, i = 1,\dots,n, , \tag{4}$$

$$0 \le z_{ij} \le \grave{z}_{ij}, i = 1,\dots,n, j = 1,\dots,n, i \neq j , \tag{5}$$

where: x_i the usable capacity (MW) at node i, $\grave{x}_i$ is the available generating capacity (MW) at node i, y_i is the load to be supplied at node i (MW), $\grave{y}_i$ is the load value at node i (MW), z_{ij} is the power flow from node i to node j (MW), $\grave{z}_{ij}$ is the transfer capability of the transmission line between nodes i and j (MW), a_{ji} is the given positive coefficients of specific losses of power when transmitted from node j to node i, $j \neq i$, $i=1,\dots,n$, $j=1,\dots,n$.

Model (1-6) is a common model of power flow for the adequacy assessment, which is solved by minimizing power shortage and is a transportation problem. The presented optimization problem is solved basically using the simplex method and the dual simplex method in their different variations, for the reason of the model simplicity. However, in [12] the authors present a valid conclusion that the model, where the power losses depend on the squared transmitted power, is a more adequate model. For this purpose, model (1-5) includes the modified balance equations, in which the constraints of type (2) are replaced with the following constraints:

$$x_i - y_i + \sum_{j=1}^{n}(1-a_{ji}z_{ji})z_{ji} - \sum_{j=1}^{n} z_{ij} = 0, \quad i = 1,\dots,n, i \neq j. \tag{6}$$

Thus, the stated problem can be presented in two forms – the problem of linear and nonlinear programming. The problem form strictly depends on the applied balance constraints in formulas (2), (6). The linear programming problem is solved if the balance constraints contain equations only with the linear losses. The nonlinear programming problem is solved, when the balance constraints contain equations with the quadratic losses. The latter can be solved by different methods of constrained and unconstrained optimization. However, this problem cannot be solved by the standard methods of unconstrained optimization because of available different equality and inequality constraints. For this purpose, the objective function and all constraints should be presented in the form of the common objective function. For example, in the YANTAR [12] software, the problem in the linear statement was solved by the Lagrange method and various modifications of the interior point method. It is worthwhile to note that the problem is solved for the long-term power

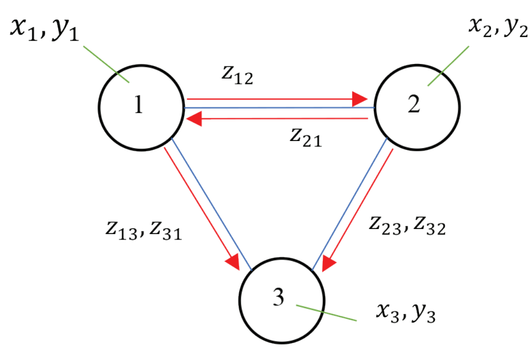

Fig. 1. Schematic representation of power flow.

system expansion planning, it uses certain equivalent methods and there is also uncertain information that can be used to solve it.

In the process of studies, we validated models (1-5) and (1), (3-6) using the test scheme (TS1). TS1 is a system (Fig. 1) consisting of three nodes and three tie lines with a ring topology. We applied a combination of the penalty function method and the gradient descent method (PFGD) as the optimization method. Based on the validation results, we obtained a solution where the optimized variables for flows Z_{12} and Z_{21} had positive values, which indicates a nonconformity of models (1-5) and (1), (3-6) to physical processes.

The obtained solution contained the data on the availability of involved flows (Table 1), whose power was distributed in both directions simultaneously, which contradicts the physical reality because each transmission line in each specific state can operate only in one direction. This fact indicates that only one variable for flows Z_{12} or Z_{21} responsible for one tie line can take a nonzero value.

Table 1. Test results, model (1) – (5), (2) – (6).

			PFGD (1)-(5)	PFGD (2)-(6)
$\grave{x}_1$	158	x_1	103,94	103,97
$\grave{x}_2$	109	x_2	105,21	105,22
$\grave{x}_3$	83	x_3	83	83
$\grave{y}_1$	91	y_1	91	91
$\grave{y}_2$	98	y_2	98	98
$\grave{y}_3$	201	y_3	102	102
$\grave{z}_{12}$	10	z_{12}	3,94	3,96
$\grave{z}_{21}$	10	z_{21}	1	1.02
$\grave{z}_{13}$	10	z_{13}	10	10
$\grave{z}_{31}$	10	z_{31}	0	0
$\grave{z}_{23}$	10	z_{23}	10	10
$\grave{z}_{32}$	10	z_{32}	0	0
		a	0,05	0,05

This situation was eliminated by the formulation of an additional constraint on power flows:

$$z_{ji}\, z_{ij}=0, i=1,\dots,n, j=1,\dots,n, i\neq j \,, \tag{7}$$

Thus, constraint (7) transforms the considered problem into a correct one from the standpoint of modeling of power flows between the nodes and alters models (1-5) and (1), (3-6) into correct ones from the standpoint of physics.

III. Models of power shortage minimization in power systems

Models (1-5), (7) and (1), (3-7) are two mathematical models of power shortage minimization in power systems, which are used for adequacy assessment. Model (1-5), (7) takes into account linear power losses, and model (1), (3-7) – quadratic power losses. However, the presented models have some downsides that have been eliminated for a long time by the modification of these models and approaches to solving the above problems.

The incorrect power flow is one of such downsides.

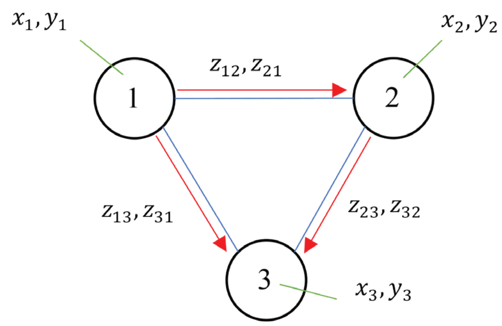

Fig. 2. An illustrative example of an incorrect power flow.

Thus, validation of models (1-5), (7) and (1), (3-7) showed the incorrect power flow but the objective function value is determined correctly and is the absolute minimum. For example, instead of power transmission by the only tie line in one direction (Fig. 2) represented by variable Z_{13} from surplus node 1 to deficient node 3, the additional node 2 and tie lines Z_{12} and Z_{23} are used for power transmission, which is not necessary at all. Such a flow "through" additional node 2 increases power transmission losses.

We applied TS1 as a tested example, optimized model (1-5), (7) using PFGD, and in addition, we arranged testing with the aid of the commercial solver of linear programming problems LP Solve. The results obtained are presented in (Table 2).

Table 2. Test results, model (1) – (5), (7).

			PFGD	LP Solve
$\grave{x}_1$	158	x_1	103,94	101
$\grave{x}_2$	109	x_2	105,21	108
$\grave{x}_3$	83	x_3	83	83
$\grave{y}_1$	91	y_1	91	91
$\grave{y}_2$	98	y_2	98	98
$\grave{y}_3$	201	y_3	102	102
$\grave{z}_{12}$	10	z_{12}	2,94	0
$\grave{z}_{21}$	10	z_{21}	0	0
$\grave{z}_{13}$	10	z_{13}	10	10
$\grave{z}_{31}$	10	z_{31}	0	0
$\grave{z}_{23}$	10	z_{23}	10	10
$\grave{z}_{32}$	10	z_{32}	0	0
		a	0,05	0,05

The study presented in [12] suggested a transformation of the objective function (1) into the form

$$\sum_{i=0}^{n} (\grave{y}_i - y_i)^2/\grave{y}_i \to \min_{y}, i \neq j. \tag{8}$$

After replacement of objective function (1) by objective function (8) in model (1-5), (7), the obtained model (1-5), (7), (8) was tested on TS1. The test resulted in the values identical to those in Table 2. The obtained solution satisfies balance constraints (2) with an error of 0.003, where ***i***=2 (which can be referred to an error of the computer calculation because of representation of numbers in the PC memory). Hence, the emerge excess flow is expressed by the variable Z_{12}, which indicates an incorrect power flow Fig. 2. The obtained values of x_1, x_2, Z_{12} do not correspond to the reference solution. The analysis of the considered example on TS1 with the constraints presented in (Table 2) shows that the additional generating capacity in an amount of 2.79 MW from node 1 is used instead of the generating capacity in an amount of 108 MW of node 2 to transmit the power in an amount of 10 MW to node 3. This additional power was transmitted to node 3 through node 2. However, the value of Z_{12} equals 2.94 MW because of the inclusion of additional power transmission losses.

Thus, a set of the obtained parameters (Table 1) satisfies the stated conditions and constraints, in particular, the determined value of the objective function minimum, but is not an optimal solution in terms of power flow. Hence, the model aimed at the correct power flow cannot eliminate the cases of incorrect power flow.

Model (3-8) with nonlinear balance constraints was tested as an experiment, and the obtained results (Table 3) were close to the previous ones (Table 2). The results of additional testing via the commercial solver GAMS (the CONOPT module for solving the nonlinear programming problems) are also presented in Table 3.

Table 3. Test results, model (3-8).

			PFGD	GAMS
$\grave{x}_1$	158	x_1	103,27	101
$\grave{x}_2$	109	x_2	105,98	108
$\grave{x}_3$	83	x_3	83	83
$\grave{y}_1$	91	y_1	91	91
$\grave{y}_2$	98	y_2	98	98
$\grave{y}_3$	201	y_3	93	93
$\grave{z}_{12}$	10	z_{12}	2,27	0
$\grave{z}_{21}$	10	z_{21}	0	0
$\grave{z}_{13}$	10	z_{13}	10	10
$\grave{z}_{31}$	10	z_{31}	0	0
$\grave{z}_{23}$	10	z_{23}	10	10
$\grave{z}_{32}$	10	z_{32}	0	0
		a	0,05	0,05

As is seen from Table 3, the results satisfy the balance constraints with an error of -0.007645 when solving (6), where i=2 (which can be referred to the error of the computer calculation because of representation of numbers in the PC memory), and the excess flow expressed by the variable Z_{12} is available (Fig. 2).

In [14] there is a statement that the quadratic component in the balance equality constraints (6) stipulates the nonconvexity of a set of feasible solutions. However, based on the data obtained in two experiments we can suppose that on the whole, the model with quadratic losses has a set of optimal solutions. This fact, in turn, influences the power flow, thereby the objective function minimum is determined correctly. In order to confirm the availability of a set of feasible solutions, we made additional calculations on TS1 with different starting points specified for the gradient descent method. The results are presented below in Table 4, where the number in the heading is the serial number of the experiment, the first column of each experiment describes the values of the starting point parameter, the second column presents the obtained solutions.

Table 4. Test results of the multi-start PFGD use.

a = 0.00009			1		2		3	
$\grave{x}_1$	158	x_1	50	103,3	100	102,9	101	104,1
$\grave{x}_2$	109	x_2	50	105,7	100	106,1	108	104,9
$\grave{x}_3$	83	x_3	50	83	100	83	83	83
$\grave{y}_1$	91	y_1	50	91	100	91	91	91
$\grave{y}_2$	98	y_2	50	98	100	98	98	98
$\grave{y}_3$	201	y_3	50	103	100	103	102,9	103
$\grave{z}_{12}$	10	z_{12}	50	2,3	100	1,8	10	3,1
$\grave{z}_{21}$	10	z_{21}	50	0	100	0	0	0
$\grave{z}_{13}$	10	z_{13}	50	10	100	10	10	10
$\grave{z}_{31}$	10	z_{31}	50	0	100	0	0	0
$\grave{z}_{23}$	10	z_{23}	50	10	100	10	0	10
$\grave{z}_{32}$	10	z_{32}	50	0	100	0	0	0

Table 4 illustrates significant variations in the generating capacity values x_1 and x_2, and also a changing resultant value of the flow z_{12}. Therewith, the values of the supplied load y_1,y_2,y_3 do not vary, which indicates that the function minimum was determined correctly. Thus, the results show the existence of a set of feasible solutions. The authors of [14] proposed some modifications to solve this problem, in particular, with the quadratic constraints. The modifications concerned both the model and the calculation scheme. To start with, the authors made an attempt to eliminate a nonconvex set of feasible solutions by transforming balance constraints (2), (6) from the equality constraints to the inequality constraints. These constraints specified fully supplied load at the node and also assumed the maximum possible power transmission, which had to resolve the problem of incorrect power flow.

$$x_i - y_i + \sum_{j=1}^{n} \left(1 - a_{ji} z_{ji}\right) z_{ji} - \sum_{j=1}^{n} z_{ij} \geq 0, \quad (9)$$
$$i = 1, \dots, n, i \neq j.$$

The application of the considered constraints on TS1 showed that the incorrect power flow remained, but the flow value z_{12} decreased, at the same time the generating capacity values x_1, x_2, x_3 changed and became equal to $\grave{x}_1$, $\grave{x}_2$, $\grave{x}_3$ respectively. However, the availability of generating

Table 5. Test results, model (1), (3-5), (7), (9).

			PFGD	GAMS
$\grave{x}_1$	158	x_1	158	101
$\grave{x}_2$	109	x_2	109	108
$\grave{x}_3$	83	x_3	83	83
$\grave{y}_1$	91	y_1	91	91
$\grave{y}_2$	98	y_2	98	98
$\grave{y}_3$	201	y_3	93	93
$\grave{z}_{12}$	10	z_{12}	0,22	0
$\grave{z}_{21}$	10	z_{21}	0	0
$\grave{z}_{13}$	10	z_{13}	10	10
$\grave{z}_{31}$	10	z_{31}	0	0
$\grave{z}_{23}$	10	z_{23}	10	10
$\grave{z}_{32}$	10	z_{32}	0	0
		a	0.05	0.05

capacity in the volume exceeding the volume needed to serve the load is physically unnatural, because this surplus capacity is blocked. For example, Table 5 displays the results obtained on TS1 using PFGD for model (1), (3-5), (9).

The authors of [14] proposed a theoretical approach to obtaining the optimal values of a solved problem with the correct power flow. The idea of the approach was to solve the problem by a two-stage optimization. The first stage suggests applying model (1), (3-5), (7), (8), obtaining an intermediate solution, and then introducing a new variable

$\grave{\Delta}_i, i = 1, \dots, n$

$$\tilde{\Delta}_i = \tilde{x}_i - \tilde{y}_i + \sum_{j=1}^{n} (1 - a_{ji}\tilde{z}_{ji})\tilde{z}_{ji} - \sum_{j=1}^{n} \tilde{z}_{ij}, \quad i = 1, \dots, n, i \neq j, \tag{10}$$

$$\sum_{i=1}^{n} \Delta_i \to \min_{\Delta}, \tag{11}$$

$$x_i + \sum_{j=1}^{n} (1 - a_{ji}z_{ji})z_{ji} - \sum_{j=1}^{n} z_{ij} - \Delta_i = \tilde{y}_i, \quad i = 1, \dots, n, i \neq j, \tag{12}$$

where $\tilde{\Delta}_i, \tilde{x}_i, \tilde{y}_i, \tilde{z}_{ji}$ is the optimal solution obtained in the first stage (10). In the second stage, the values of $\tilde{y}_i$ were fixed and a new objective function of form (11) and balance constraints of type (12) were introduced, subsequently the problem was solved for the variables Δ_i, x_i, z_{ji}, and the model took form (3-5), (7), (9), (11), (12), $j \neq i$, $i=1,\dots,n$, $j=1,\dots,n$.

The proposed modifications were validated on TS1 using PFGD. The values of the variables obtained in the first stage of the problem-solving process are presented in (Table 5). However, in the second optimization stage, the results did not change, which indicated that the results of this model were incorrect.

We propose the following algorithms to deal with incorrect power flow. The two-stage optimization must be applied in a different way: the power shortage minimization problem for model (1), (3-5), (7), (9) must be solved in the first stage. Such an approach will provide a convex set of feasible solutions. Then, the obtained optimal solutions for the variable y_i must be fixed and a new variable must be denoted as $\grave{y}_i$. Subsequently, in the second stage, it is necessary to generate a new objective function, which is the minimization of the second Euclidean norm for all the flows:

$$\sum_{i=1}^{n} {z_{ji}}^2 \to \min_{z}, i \neq j, \tag{13}$$

and also to transform the current balance constraints (10) into the balance constraints presented below:

$$x_i - \tilde{y}_i + \sum_{j=1}^{n} (1 - a_{ji}z_{ji})z_{ji} - \sum_{j=1}^{n} z_{ij} = 0, \quad i = 1, \dots, n. \tag{14}$$

The performance of the approach of the sequential two-stage optimization and the interaction of models (1), (3-5), (7), (9) and (4-5), (7), (13), (14) was validated on TS1 using PFGD. The results of the first stage are presented in Table 5, the parameters obtained in the test of the second stage are indicated in Table 6.

Table 6. Test results, model (4-5), (7), (13), (14).

			PFGD	GAMS				PFGD	GAMS
$\grave{x}_1$	158	x_1	99,5	101	$\grave{x}_1$	158	x_1	100,98	101
$\grave{x}_2$	109	x_2	106,5	108	$\grave{x}_2$	109	x_2	107,98	108
$\grave{x}_3$	83	x_3	83,11	83	$\grave{x}_3$	83	x_3	83	83
$\grave{y}_1$	91	y_1	91	91	$\grave{y}_1$	91	y_1	91	91
$\grave{y}_2$	98	y_2	98	98	$\grave{y}_2$	98	y_2	98	98
$\grave{y}_3$	93	y_3	93	93	$\grave{y}_3$	103	y_3	102,98	103
$\grave{z}_{12}$	10	z_{12}	0	0	$\grave{z}_{12}$	10	z_{12}	0	0
$\grave{z}_{21}$	10	z_{21}	0	0	$\grave{z}_{21}$	10	z_{21}	0	0
$\grave{z}_{13}$	10	z_{13}	8,49	10	$\grave{z}_{13}$	10	z_{13}	9,98	10
$\grave{z}_{31}$	10	z_{31}	0	0	$\grave{z}_{31}$	10	z_{31}	0	0
$\grave{z}_{23}$	10	z_{23}	8,49	10	$\grave{z}_{23}$	10	z_{23}	9,98	10
$\grave{z}_{32}$	10	z_{32}	0	0	$\grave{z}_{32}$	10	z_{32}	0	0
		a	0,05	0,05			a	0,00009	0,00009

Table 7. Test results, model (4-5), (7), (14), (15), PFGD.

	Experiments				
	1	2	3	4	5
$\grave{x}_1$	158	150	100	250	158
$\grave{x}_2$	109	150	150	140	109
$\grave{x}_3$	83	100	200	100	83
$\grave{y}_1$	91	150	100	150	91
$\grave{y}_2$	98	250	150	150	98
$\grave{y}_3$	201	150	100	200	201
$\grave{z}_{12}$	10	50	50	50	10
$\grave{z}_{21}$	10	50	50	50	11
$\grave{z}_{13}$	10	50	50	50	12
$\grave{z}_{31}$	10	50	50	50	13
$\grave{z}_{23}$	10	50	50	50	14
$\grave{z}_{32}$	10	50	50	50	15
a	0,05	0,05	0,05	0,05	0,05
	1	2	3	4	5
x_1	100,43	150	100	168,58	100,43
x_2	107,43	150	150	140,01	107,43
x_3	83,02	100	100	100,01	83,02
y_1	91	150	100	150	91
y_2	98	150	150	145	98
y_3	93	100	100,01	105	93
z_{12}	0	0	0	9,29	0
z_{21}	0	0	0	0	0
z_{13}	9,43	0	0	9,29	9,43
z_{31}	0	0	0	0	0
z_{23}	9,43	0	0	0	9,43
z_{32}	0	0	0	0	0

The results in Table 6 show that the parameters obtained with this combination of models are adequate and close to the target values of the commercial solver GAMS. The more exact and closer results were obtained by modification of the objective function of model (13) of the second stage that is given below:

$$\sum_{i=1}^{n} x_i \rightarrow \min_{x}. \tag{15}$$

The objective function (16) is intended for minimization of generating capacity, which can yield a positive economic effect in power generation and distribution. Model (4-5), (7), (14), (15) with this objective function for the second optimization stage was validated on TS1 and showed closer and more valid results than model (4-5), (7), (13), (14). The calculation results are illustrated in Table 6, the values of the constraints are presented in the upper part and the results are given in the lower part.

IV. Methods for power shortage minimization in power systems

The penalty function method [15] can be applied to optimization problems with different types of constraints. This method makes it possible to transform the initial problem with constraints into the problem that can be solved by the unconstrained optimization methods. Such a transformation allows the use of simpler methods for solving the linear and nonlinear programming problems and the increase in calculation accuracy with the correct selection of parameters. The main changes occur in the objective function, to which the constraints in the form of penalty functions are added. Thus, changes in the system can lead to automatic involvement of the penalty function, whose value will sharply rise. In this case, the response to the penalty will be controlled by the optimization method, and finally, the function will be directed to a sought solution. There are two subtypes of the penalty function method – internal and external penalty functions.

Further, we will consider the external penalty function method, since this method allows solving the constrained optimization problems with both equality and inequality constraints. In the general form, the function and constraints look as follows:

$$f(x) \rightarrow min, \tag{16}$$

subject to equality and inequality constraints:

$$\varphi_i(x) = 0, i = 1, \ldots, I, \tag{17}$$

$$g_j(x) \leq 0, j = 1, \ldots, J, \tag{18}$$

The strategy of a search for the optimal solutions suggests that in this method the penalty functions *Φ(x,γ)* are chosen so that their values are equal to zero inside and on the boundary of the feasible region *G*, while beyond the region they are positive and increase the more, the higher the violations of the constraints. Thus, here the distance from the feasible region *G* is "penalized".

As a rule, the function:

$$\Phi(x,\gamma) = \frac{\gamma}{2} \tag{19}$$

where:

$$max\left(0, g_j(x)\right) = \begin{cases} 0, g(x) \leq 0 \\ g_j(x), g_j(x) > 0 \end{cases} \tag{20}$$

is applied as an external penalty function.

The auxiliary function *F(x,γ)*, in this case, takes the form:

$$F(x,\gamma) = f(x) + \Phi(x,\gamma)\,. \tag{21}$$

The starting point for search is usually specified beyond the feasible region *G*. The point x^* (γ^k) of the unconstrained minimum of the auxiliary function $F(x^*,\gamma^k)$ for x with the specified parameter γ^k is searched with the help of any method of constrained optimization (of the zero, first or second order) at each k-th iteration. The obtained point x^* (γ^k) is used as a starting one at the next iteration with the increasing value of the penalty parameter. With the unlimited growth of γ^k the sequence x^* (γ^k) converges to the constrained minimum point x^*.

V. Application of the penalty function method to power shortage minimization problem

We apply the described method to transform the stated constrained optimization problem with the balance constraints subject to the quadratic losses. Then, we generate an auxiliary function by integrating the constraints in a required format into the external penalty function for model (1), (3-6), (8), (10), which will look as follows:

$$\Phi(x, y, z, a, \dot{x}, \dot{y}, \dot{z}, \gamma) = \frac{\gamma}{2} \tag{22}$$

The value of the penalty multiplier is controlled by the parameter γ. In this problem, the penalty is gradually increased by an order of magnitude per iteration, the initial value of this parameter is equal to 10. The remaining penalty function parameters are correlated in accordance with the

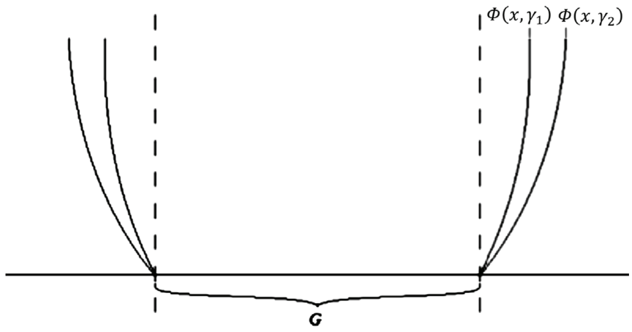

Fig. 3. Graph of the strategy of search (with $\gamma_2 > \gamma_1$).

available constraints, where the equality constraints are the penalty function parameters:

$$\varphi_i(z) = z_{jk} * z_{kj}, \quad i = 1, \dots, I, j = 1, \dots, n, k \neq j \tag{23}$$

The inequality constraints are:

$$max\left(0, t_j(x, y, z, a)\right) = \begin{cases} 0, t_j(x, y, z, a) \leq 0 \\ t_j(x, y, z, a), t_j(x, y, z, a) > 0 \end{cases}, \quad j = 1, \dots, n, \tag{24}$$

$$t_j(x, y, z, a) = x_i - y_i + \sum_{k=1}^{n}(1 - a_{ki}z_{ki})z_{ki} - \sum_{k=1}^{n} z_{ik} \tag{25}$$
$$j = 1, \dots, n, k = 1, \dots, n, i = 1, \dots, n, k \neq i,$$

$$max\left(0, g_j(x)\right) = \begin{cases} 0, g(x) \leq 0 \\ g_j(x), g_j(x) > 0 \end{cases}, \quad j = 1, \dots, n, \tag{26}$$

$$max\left(\grave{x}, g_j(x)\right) = \begin{cases} \grave{x}, g(x) \geq \grave{x} \\ g_j(x), g_j(x) < \grave{x} \end{cases}, \quad j = 1, \dots, n, \tag{27}$$

$$max\left(0, g_j(y)\right) = \begin{cases} 0, g(y) \leq 0 \\ g_j(y), g_j(y) > 0 \end{cases}, \quad j = 1, \dots, n, \tag{28}$$

$$max\left(\grave{y}, g_j(y)\right) = \begin{cases} \grave{y}, g(y) \geq \grave{y} \\ g_j(y), g_j(y) < \grave{y} \end{cases}, \quad j = 1, \dots, n, \tag{29}$$

$$max\left(0, g_j(z_{ki})\right) = \begin{cases} 0, g(z_{ki}) \leq 0 \\ g_j(z_{ki}), g_j(z_{ki}) > 0 \end{cases}, \quad j = 1, \dots, n, k = 1, \dots, n, i = 1, \dots, n, k \neq i \tag{30}$$

$$max\left(\grave{z}_{ik}, g_j(z_{ik})\right) = \begin{cases} \grave{z}_{ik}, g(z_{ik}) \geq \grave{z}_{ik} \\ g_j(z_{ik}), g_j(z_{ik}) < \grave{z}_{ik} \end{cases}, \quad j = 1, \dots, n, k = 1, \dots, n, i = 1, \dots, n, k \neq i. \tag{31}$$

Further, we replace the objective function f(x), and also penalty functions (23) and (26) in accordance with the replaced balance constraints by (15), however, (24), (27) – (32) are not subject to changes. In the end, the penalty functions must be of the following form:

$$\Phi(x, \tilde{y}, z, \grave{x}, \grave{y}, \grave{z}, \gamma) = \frac{\gamma}{2} \tag{32}$$

where:

$$\varphi_i\left(x, \tilde{y}, z_{ji}, a_{ji}\right) = x_i - \tilde{y}_i + \sum_{j=1}^{n}\left(1 - a_{ji}\right)z_{ji} - \sum_{j=1}^{n} z_{ij}, i = 1, \dots, n. \tag{33}$$

VI. Conclusion

The assessment of power system adequacy is topical and necessary for power system expansion planning. The power shortage minimization problem is solved within the system adequacy assessment by the Monte Carlo method. The paper presents an analysis of the existing power shortage minimization models. The study revealed some downsides of the models. A case study demonstrates that the models have a nonconvex set of feasible solutions. The paper describes a technique for transformation of balance constraints for modeling of the problem with a set of feasible solutions. Consideration is also given to different modifications of the power shortage minimization models. Based on the studies performed we propose two modifications of the models.

The stated problems were solved by the gradient descent method. The commercial solvers GAMS (CONOPT) and LP Solve were applied to obtain reference solutions.

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