Modeling of Decreasing Short-term Marginal Costs and Corresponding Supply Functions of Condensing Power Plants at a Day-Ahead Electricity Market

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***Abstract* — The paper presents a calculation and an analysis of short-term marginal costs and corresponding supply functions of a condensing power plant. The calculation can be applied in power plant control systems or bidding support software to improve plant efficiency at the day-ahead market. A specific turbine is considered. Mathematical modeling is applied to determine first the short-term marginal costs and then the supply function of a price-taker based on the energy unit energy characteristics. The analysis shows that the short-term marginal costs function of a unit can decrease or can have decreasing segments. In this case, the supply function of a price-taker is not the same as the short-term marginal costs function. It is also shown that the supply function can be undefined for the output below the minimum output of the unit as well as within the range of decreasing short-term marginal costs. The form of the supply function does not correspond to the amount of units in operation.**

***Index terms* — thermal power plants; day-ahead market; supply function, marginal costs, market bidding**

I. Introduction

Relationship between marginal costs and supply functions of a price-taking producer is a key element of the modern market design. Generation is considered to be a competitive industry and according to the concept of perfectly competitive market, an individual supply function is the same as the marginal costs function under perfect competition. Otherwise, the producer can be considered to abuse market power. Some other often supposed properties of marginal costs functions are:

- the functions are defined from 0 to maximum capacity of the unit (or maximum output of the unit within the considered period);
- the functions are non-decreasing (marginal costs at higher output values cannot be lower than those at lower values);
- the functions do not depend on market clearing condition.

Many papers were published and models were developed based on these assumptions. Unfortunately, researchers pay very little attention to the assumptions themselves. Section II of the paper gives a literature review of how researchers deal with them.

Section III is focused on the calculation of a short-term supply function of an energy unit based on its energy characteristics. The turbine K-800-23,5-3 is considered in both single-boiler-single-turbine and two-boiler-single-turbine arrangements, and the former one operates within one-unit and four-unit power plants.

Section IV calculates supply functions based on the short-term marginal cost functions. Section V concludes the paper, describes some possible implications and suggests future study topics.

II. Literature review

The calculation of short-term marginal costs of real energy units or plants based on their measured characteristics is a rare focus of scientific research.

The author of [1], for example, states that the supply curve of a price-taking supplier is the same as his marginal costs curve. The author describes the supply function as a horizontal or slightly slanting line within the range of the unit installed capacity, and as a vertical line at the end of the unit installed capacity. The supply function itself is not calculated from the turbine and boiler characteristics.

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In [2], the supply function is introduced with reference to a typical short-term costs function, which is continuously differentiable and convex due to the law of diminishing marginal utility. The supply function is defined within the range from 0 to the maximum unit capacity, it is monotonically increasing and continuous.

In [3], the authors define marginal costs as a derivative of full costs function with respect to production value. In the examples, the marginal costs are given in cents per kWh irrespective of unit load factor (non-decreasing). The minimum output of units is not taken into account. In many other studies the cost functions of generators are assumed to be convex [4], [5].

For efficient market bidding support, the control systems must use the power plant optimization and unit energy characteristics. Unfortunately, very few researchers really do it. In [6]-[10], for example, the authors calculate costs based on the plant operating conditions, but they do not make a conclusion concerning supply functions. Therefore, calculation of supply functions of price-taking producers from their internal optimization and unit energy characteristics is an important but poorly investigated problem.

III. Short-term marginal costs

The scope of the paper is limited to the short-term marginal costs (STMC). These costs are typical of the decision whether or not to increase or decrease power output by 1 MW. The plant itself is constructed and the salary and taxes are already paid and are not taken into account. The costs that vary with the power output (mainly fuel cost) are under consideration. This approach is related to short-term markets (a day-ahead spot market and a balancing market).

A. STMC Functions of a Unit and their Domain of Definition

For most boilers used in Russia the minimum output is 40-60% of the installed capacity. Since the boilers cannot generate less than this value, the domain of definition of STMC functions starts at the minimum output and ends at the installed capacity of the unit for the single boiler-single turbine arrangement.

The dependence of the main steam flow rate (D_0, tons of steam per hour) on electrical output (N) of the turbine K-800-23,5-3 is best approximated with the equation taken from [11], (p. 273, Fig. 3.42.a)

$$D_0 = 3.271\,N - 81.379. \qquad (1)$$

The dependence of the boiler fuel consumption (B, ton of fuel per ton of main steam) on the boiler load factor (U_b) is assumed to be

$$B = 0.191\,U_b^{\,-0,059}, \qquad (2)$$

where U_b takes the values between 0.4 and 1. In the single-boiler-single-turbine arrangement

$$N = 800\,U_b. \qquad (3)$$

From (1)-(3), hourly fuel costs F given fuel price (C_f, RUB/t) are

$$F = C_f D_0 B = C_f\,(0.927\,N^{0,941} - 23.058\,N^{-0,059}) \qquad (4)$$

and the STMC function is

$$\mathrm{C}_{\mathrm{stm1\text{-}1}} = dF / dN = C_f\,(0.872\,N^{-0,059} - 1.360\,N^{-1,059}) \qquad (5)$$

The calculated STMC function for the K-800-23,5-3 turbine in condensing mode is shown in Fig. 1. The minimum output is 40% and the maximum output is 100% of the installed capacity. The turbine energy characteristic is non-linear. The efficiency of the turbine rises with the output. The boiler efficiency also increases as it approaches the rated steam-output capacity (see, for example, [12], p. 197, Fig. 7.2.b). Therefore, the STMC function of the unit is decreasing. The fuel price is assumed to be C_f = 1200 RUB/t.

For the two-boiler-single-turbine arrangement, the STMC function domain of definition is larger and can start with 20-30% of the unit installed capacity because the boilers are put into operation one by one as the load increases. The discontinuity, however, arises at the point where one boiler operating at full capacity switches to two boilers operating at half capacity.

Let us consider the same turbine with two boilers of twice lower capacity but with the same characteristics, i.e. (1)-(2) are true. Instead of (3), we assume

$$N = 400\,U_b \qquad (6)$$

for the case where one boiler is in operation and the other one is out of operation. Therefore, from (1)-(2) and (6) for the turbine load factor U_t = 0.2...0.5 the STMC function is

$$C_{\mathrm{stm2\text{-}1}} = C_f\,(0.837\,N^{-0,059} - 1.306\,N^{-1,059}), \qquad (7)$$

and for the turbine load factor U_t = 0.4...1 (both boilers are in operation and are loaded simultaneously, i.e. their output increases concurrently) STMC function is similar to (5)

$$C_{\mathrm{stm2\text{-}2}} = C_f\,(0.872\,N^{-0,059} - 1.360\,N^{-1,059}). \qquad (8)$$

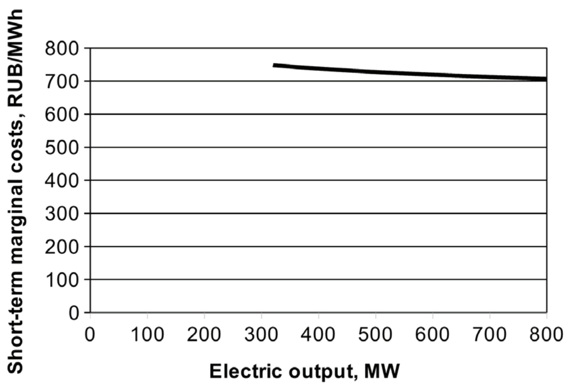

Fig. 1. The STMC function for the K-800-23,5-3 turbine in condensing mode.

The STMC function for the two-boiler-single-turbine arrangement is shown in Fig. 2. The minimum output is 20% and the maximum output is 100% of the installed capacity. It is worth noting that the capacity in the range of 40-50% can be maintained both by one or two boilers with different costs.

B. STMC Function of a Plant

As an example, we consider a plant with 4 similar single-boiler-single-turbine units described earlier. The minimum output of each unit is assumed to be 60% of the unit installed capacity. The STMC function is calculated assuming base-load condition of the plant. This means that no changes in the unit mix during operation are considered. Since the units are similar, four combinations of units involved are possible: one, two, three or four units in operation. The STMC function for n similar units involved is calculated as

$$C_{\text{stmn}} = C_f\,(0.872\,(N_p/n)^{-0.059} + 1.360\,(N_p/n)^{-1.059}), \quad (9)$$

where N_p – electric output of the power plant (units increase output simultaneously). The calculated STMC function for the power plant with four K-800-23,5-3 turbines in condensing mode is shown in Fig. 3. It is calculated assuming simultaneous loading of the units involved. Consecutive loading will cause a different result.

It is worthwhile to mention that the output ranges 0...480 MW and 800...960 MW go beyond the feasibility region for the plant under the assumptions given. On the other hand, the ranges 1440...1600 MW and 1920...2400 MW can be maintained by two different unit sets with different costs.

IV. STMC-based supply functions

Under perfect competition, any market participant is a price-taker, i.e. they are unable to influence the market clearing price. Generally, the supply curve of a generator is a set of points in the coordinate plane Price (Output), where each output value provides the maximum profit at a corresponding price.

If the STMC function is non-decreasing, as it is usually assumed, the Profit (Output) function at each price given has one extremum, and the supply function coincides with the STMC function. For decreasing STMC functions or those with decreasing segments, it is not the case. The full algorithm should be applied: assume a market price C_e, find the output value which provides maximum revenue minus fuel costs of the supplier, take the pair price-output as one point of the supply curve, and repeat the same procedure for other market prices assumed. The optimization criterion for the above described plant with n units (single-boiler-single-turbine arrangement, parallel loading) can be written as

$$\max\,(n, N_p) = (C_e\,N_p - F\,(n, N_p)), \quad (10)$$

where

$$F\,(n, N_p) = nC_f\,(0.927(N_p/n)^{0.941} - 23.058\,(N_p/n)^{-0.059}). \quad (11)$$

An additional constraint should be taken into account for decreasing STMC, i.e. at any point of the supply function the revenue cannot be less than the short-term costs. In other words, if the maximum revenue at a certain market price does not cover even the STMC, the price should be excluded from the feasible region of the supply function since the negative short-term profit does not encourage the producer to generate

$$\max\,(n, N_p) = (C_e\,N_p - F\,(n, N_p)) > 0. \quad (12)$$

For non-decreasing STMC, the condition is met naturally.

Figure 4 demonstrates the day-ahead market supply function of a power plant with one turbine K-800-23,5-3 by dotted line, and the plant with four turbines – by solid line. The function of the 4-unit plant is calculated under static condition, i.e. assuming the same power output and the same number of units involved. No starts or stops of units are considered. It is worth emphasizing that:

- the supply function consists of one vertical segment

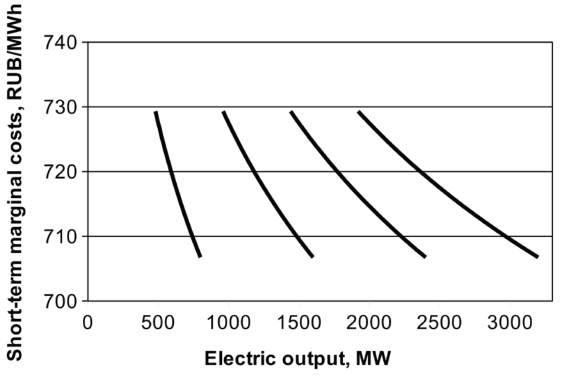

Fig. 2. The STMC function for the power plant with four K-800-23,5-3 turbines in condensing mode.

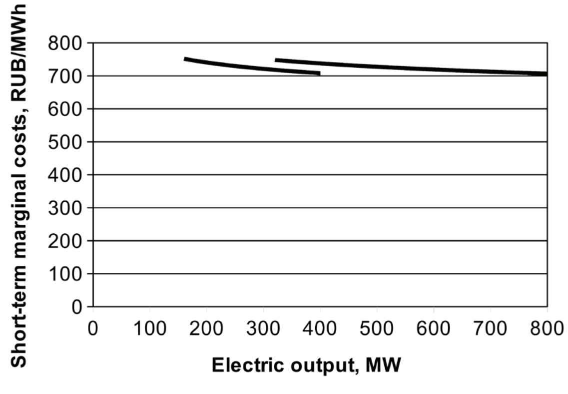

Fig. 3. The STMC function for the two boilers-single turbine arrangement.

both for 1-unit and 4-unit plant, the amount of segments does not depend on the amount of units;

- there are no horizontal or slanting lines or segments in both supply functions;
- both the 1-unit plant and the 4-unit plant do not generate at prices below 726.55 RUB/MWh and switch to maximum output at higher prices;
- the 4-unit plant never operates 1, 2 or 3 units and it is a perfectly competitive behavior. Both considered producers are never interested in operation with partial load.

Calculation of a dynamic supply function of a 4-unit plant requires additional assumptions: initial condition (n_0, N_{p0}), costs of a unit start (Fs) and the number of hours the started units will operate (t). The optimization is performed according to (10)-(12) for the same set of units and for the units that are switched on and off. Switching on units causes additional costs, and the optimization criterion (10) transforms to

$$\max(n, N_p) = (C_e N_p - F(n, N_p) - ((n - n_0)F_s) / t), \quad (13)$$

The dynamic supply function of the 4-unit plant assuming $n_0 = 1$, F_s = 5000 RUB, t = 8 hours is shown in Fig. 5.

The plant operates one or four units depending on the market price. Intermediary sets of 2 and 3 units in operation are never an optimal solution.

V. Conclusions and implications

The paper describes short-term marginal costs of an energy unit and calculates supply functions for the plants with one such a unit and four identical units. The study shows that:

- the short-term marginal costs function of a unit can be decreasing or can have decreasing segments;
- if the short-term marginal costs function is decreasing or has decreasing segments the corresponding supply function differs from the marginal costs function;
- for the range below the minimum output, the short-term supply function is undefined; it is also undefined within the range of decreasing short-term marginal costs;
- the amount of segments of supply function under perfect competition does not depend directly on the amount of units;
- dynamic short-term supply function of a plant differs from the static one because of initial condition, costs of a unit start and the number of hours the started unit is expected to run.

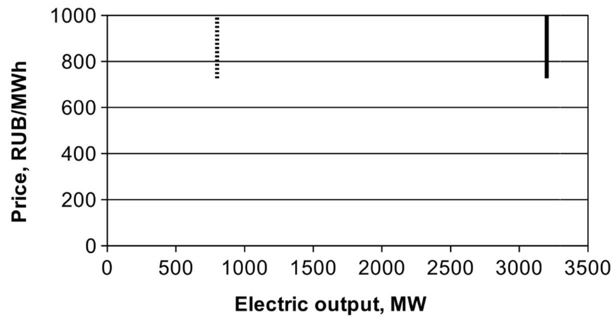

Fig. 4. The day-ahead spot market supply function of a power plant with one turbine (dotted line) and with four turbines (solid line) K-800-23,5-3.

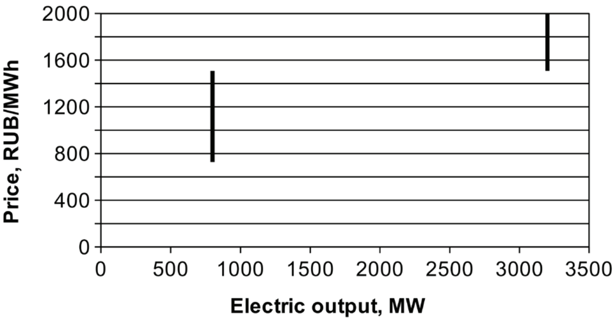

Fig. 5. The dynamic supply function of a 4-unit plant.

References

[1] Stoft S. "Power System Economics: Designing Markets for Electricity". *IEEE Wiley*, p. 496, 2002.

[2] Kirschen D.S., Strbac G. "Fundamentals of Power Systems," *Economics. John Wiley Sons, Ltd*., p. 284, 2004.

[3] Schweppe F.C., Caramanis M.C., Tabors R.D., Bohn R.E. "Spot Pricing of Electricity,". *Kluwer academic publishers,* p. 356, 1988.

[4] Aizenberg N. "Application of Supply Function Equilibrium Model to Describe the Interaction of Generation Companies in the Electricity Market,". *Lecture Notes in Computer Science,* vol. 9869, pp. 469-479, 2016.

[5] Palamarchuk S.I. "Strategic Behavior of Generation Companies in the Generation Scheduling Process. Environment and Electrical," *Engineering: Proceedings of the 2015 IEEE 15th International Conference.* Rome, Italy. pp. 192-197.

[6] A.M. Kler, A.S. Maksimov, A.V. Chalbyshev, E.L. Stepanova, Tasks of "Internal State Optimization of CHP at the day-ahead market," *Energetik*, vol. 8, pp. 20-23. 2015. (in Russian.)

[7] A.M. Kler, A.S. Maksimov, A.V. Chalbyshev,"Optimization of Operation Modes of Organic Fuel Energy Sources Taking into Account," *Electricity Wholesale Market Conditions* Voronezh State Technical University Bulletin, vol. 1, pp. 73-79. 2013, (in Russian.)

[8] A.M. Kler, A.S. Maksimov, A.V. Chalbyshev, E.L. Stepanova, "Optimization of CHP Operation Modes for Profit Maximization in Balancing Electricity Market," *Proceedings of the Russian Academy of Sciences. Power Engeneering*, vol. 2, pp. 71-80.

2014, (in Russian.)

[9] A.M. Kler, A.S. Maksimov, A.V. Chalbyshev, E.L. Stepanova, “Choice of Optimal Unit Mix and Operation Mode of a CHP at a Day-Ahead Market,” *Proceedings of the Russian Academy of Sciences. Power Engeneering*, vol. 4. pp. 116-129, (in Russian.)

[10] A.M. Kler, A.S. Maksimov, A.V. Chalbyshev, E.L. Stepanova, “Optimal Dispatching of CHP Units at a Day-Ahead Market,” *Siberian Journal of Science*, vol. 15 (special issue), pp. 63-67. 2015, (in Russian.)

[11] Thermal and nuclear plants: Handbook Edited by member-corr. of RAS A.V. Klimenko and prof. V.M. Zorin. 3rd edition - Moscow, p. 648, (Heat power engineering and thermal engineering; Book 3,) (in Russian.)

[12] Yu.M. Lipov, Yu.M. Tretjakov “Boiler plants and steam generating units,” Moscow-Izhevsk, *Scientific Publishing Center Regular and Chaotic Dynamics*, p. 592, 2003, (in Russian.)

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