Locational Marginal Pricing in Multi-Period Power Markets

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***Abstract* — The objective of this paper is to develop an analytical framework for interpretation of locational marginal prices (LMPs) in multi-period power markets with intertemporal ramping, limited energy, and energy storage constraints. Previous research dedicated to the techniques for decomposition of LMPs explicitly shows their formation as a spatial structure of components due to power flow, transmission and voltage constraints. In contrast to the traditional point of view, this study proposes formulae for discussing a temporal LMP structure, where LMPs are obtained as Lagrange multipliers for nodal real power balances in a multi-period AC optimal power flow (OPF) problem. In the beginning, marginal resources are discussed. It is shown that an energy resource with unbounded output at a specific time period may not be marginal. Then, the resources that actually form LMPs in the energy system are determined. The study shows that not all marginal resources directly affect LMPs. Finally, the dependence of LMPs on marginal resources from different time periods is considered. It is shown that binding ramping constraints lead to "cardiogram" curves of LMPs, while limited energy and energy storage constraints smooth them out and are used to form LMPs based on the overall price situation in specific time periods. The aim of the methodology is not to determine LMPs but to identify contribution of particular constraints that affect their formation. The methodology has been tested on the IEEE-30 energy system extended with a daily load profile for a day-ahead market with a full AC OPF model.**

***Index Terms* — Locational marginal prices, multi-period power market, ramping rates, energy storage.**

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NOMENCLATURE

Sets and indices:

$g \in \mathcal{G}$	Generators
$d \in \mathcal{D}$	Demands
$i \in \mathcal{N}$	Nodes
$s \in \mathcal{S}$	Storage resources including generating $g = s$ and demanding $d = s$ resources
$r \in \mathcal{R}$	Different resources like generators, demands, or storage
$t \in \mathcal{T}$	Time periods
t_{start}	Initial time period ($t_{start} \notin \mathcal{T}$)
t_{end}	Last time period $t_{end} \in \mathcal{T}$

Parameters:

$\overline{E}_g, \underline{E}_g$	Maximum and minimum energy limits for generator g (MWh)
$C_{g,t}, C_{d,t}$	Offer cost of generator or storage g and bid cost of demand or storage d at time period t (rub/MWh)
R_g^+, R_g^-	Maximum ramp-up and ramp-down rates for generator g
$\overline{P}_d, \underline{P}_d$	Maximum and minimum real power output levels for demand or storage d (MW)
$\overline{P}_g, \underline{P}_g$	Maximum and minimum real power output levels for generator or storage g (MW)
$\overline{SC}_s, \underline{SC}_s$	Maximum and minimum state of charge levels for storage s (MWh)
$Q_{d,t}$	Reactive power demand d at time period t (MVAr)
$\overline{Q}_g, \underline{Q}_g$	Maximum and minimum reactive power output levels for generator or storage g (MVAr)
$\eta_s \leq 1$	Storage s storing efficiency
$\eta_d \leq 1$	Storage d charging efficiency
$\eta_g \geq 1$	Storage g discharging efficiency
Δt	Time period duration (hr)

Optimization variables:

$SC_{s,t}$	State of charge for storage s at time t (MWh).

$P_{g,t}, P_{d,t}$	Real power output levels for generator or storage g and demand or storage d at time period t (MW).
$Q_{g,t}$	Reactive power output levels for generator or storage g at time t (MW)
X_t	Vector of voltage phases and magnitudes at time t (deg, p.u.)

Dual variables:

λ_t^{node}	Vector of Lagrange multipliers associated with nodal power balance constraints for real (which define LMP) and reactive power at time period t.
$\lambda_t^{tc}, \lambda_t^{vc}$	Vector of Lagrange multipliers associated with transmission and voltage constraints at time period t
$\nu_{g,t}^{+}, \nu_{g,t}^{-}$	Lagrange multipliers associated with intertemporal ramping constraints for generator g at time period t
$\overline{\nu}_g^E, \underline{\nu}_g^E$	Lagrange multipliers associated with intertemporal limited energy constraints for generator g
$\nu_{s,t}$	Lagrange multipliers associated with storage s' state of charge at time period t
$\overline{\pi}_{g(d),t}, \underline{\pi}_{g(d),t}$	Lagrange multipliers associated with maximum and minimum active power output levels
$\overline{\pi}_{s,t}^{SC}, \underline{\pi}_{s,t}^{SC}$	Lagrange multipliers associated with state of charge constraints for storage s at time period t

Other variables:

B_j	Cost component of resource j (storage s or other resources) in the social benefit function.
$J_t^{node}, J_t^{tc}, J_t^{vc}$	Jacobian matrices in respect to voltage phases and magnitudes X for nodal power balance, transmission, and voltage constraints at time period t
$\Delta\lambda_t^{f}, \Delta\lambda_t^{tc}, \Delta\lambda_t^{vc}$	LMP components at time period t due to power flow, transmission and voltage constraints.

I. Introduction

Decentralized electricity sectors in many parts of the world allowed raising competition among owners of power generating equipment. Nodal pricing of electricity through locational marginal prices (LMPs) gives incentives to safely manage an energy system characterized by congestion and dependence on detached behavior of different stakeholders [1, 2]. Being one of the most powerful tools of congestion management, LMPs are implemented in many electricity markets all over the world, including Russia, the United States, New Zealand, Singapore, etc.

The LMP at a particular node is defined as a response of the system in the form of an increased cost called marginal cost to an incremental increase in a demand at that particular node while respecting all the security constraints of the system. Disparate locations of nodes lead to different marginal costs due to distinctive losses, diverse influence of transmission and other constraints.

There has been a very significant research effort undertaken internationally since the 1990s to understand how LMPs are derived, how to interpret and decompose them as understandable components. A classical approach allows decomposing each LMP into energy, loss and congestion components [3, 4, 5]. This approach is still viable in present days but needs to reflect the greater perception of the role of marginal generators [6, 7].

Generation scheduling meets different multi-period constraints, the most known of which are limited energy, energy storage and ramping constraints.

Traditional energy storage systems imply hydropower reservoir systems [8, 9], which could include either common or pumped-storage hydro plants. Water reservoir constraints are controlled during daily hydrothermal generation scheduling [10, 11]. Similar to limited energy hydro power plants, some thermal power plants need to be rescheduled while meeting fuel constraints [12].

Pumped hydro energy storage provides various services and contributions to the power system including load leveling, maintenance of voltage and stability in a system, generating capacity, etc. Using pumped hydro energy storage allows reduction in overall energy system costs and CO2 emissions. Currently, the co-use of pumped hydro energy storage in power markets usually causes trouble because of the necessity of predicting charging and discharging windows, and can be improved [13].

For pumped-storage hydropower plants, as well as other storage systems, the Federal Energy Regulatory Commission in the USA tries to remove legislature barriers to the participation of electric storage resources in the capacity, energy, and ancillary service markets operated in the USA [14, 15]. A great interest is dedicated to the level of models and algorithms of OPF with energy storage [16, 17]. Apart from direct market implementation, there are propositions of financial storage rights [18, 19] and, what is more, the idea of electric vehicles as electricity storage devices may be relevant [20, 21].

According to [22, 23, 24], energy storage technologies are still too expensive for full deployment to benefit power markets, although, technological advance in different types of storage makes it rational to pre-define a full-fledged participation as an independent resource in economic dispatch and market operations.

High penetration of renewable resources with its inherent variability dictates new unconventional methods of energy management. The famous transformation of daily load shape into a "duck curve" in California power market requires advanced ramping capacity to handle sharp conventional power generation changes [25, 26]. The need for an exact model of ramping constraints arose. In order to provide correct price signals, authors of [16, 27] propose new pricing models for real-time markets, while in [28, 29] ramping products, namely, additional payments for generators selected to provide ramping capability are discussed.

In the Russian power market [30], there are ramping constraints and limited energy constraints. The former occur mainly in the European region, the latter are relevant for hydro plants in Siberia and Volga regions. While energy storage constraints are not incorporated into market procedures, there are bulk energy storage resources like the Zagorskaya pumped-storage hydro plant with a 1200 MW generation capacity, 1320 MW charging capacity, and a yearly generation of 1 900 million kWh [31]. The goal of the paper is relevant for the Russian power market since the existing intertemporal constraints affect LMPs in day-ahead and balancing markets and the prospect of implementing energy storage constraints may benefit as in the case of reducing overall energy system costs.

LMPs as Lagrange multipliers in a multi-period OPF incorporate intertemporal constraints along with transmission and voltage constraints. This brings on new issues in the field of LMP interpretation. The first issue lies in the ambiguity of a status of a marginal generator or, more generally, a marginal resource. In [16], a resource is called marginal when its output is not bounded with the corresponding instantaneous maximum and minimum output level constraints. In [32], on the contrary, all corresponding constraints should be non-binding to call a resource marginal. In order to study LMPs and their interpretation we need to examine the marginal status of a resource by means of the Karush-Kuhn-Tucker (KKT) necessary conditions and possibility of responding to the market needs. The second issue arises with interconnection of time periods when a marginal resource in one period affects the LMPs in other periods. In conclusion, we can infer that the temporal formation of LMP, in contrast to its one-period spatial structure, is studied insufficiently.

This paper attempts to measure LMPs under intertemporal constraints and bridge the gap in the LMP interpretation in the multi-period OPF. The objectives of this study are (a) to analyze the economic impacts of current multi-period ramping and energy constraints; (b) to introduce a new product of energy storage in the optimization problem; and (c) to explore the conditions affecting the LMP by different intertemporal constraints. While meeting the objectives, we consider a day-ahead market and a simulated market clearing process of a scaled system in the full AC OPF framework.

This approach differs from the previous studies in that a) a full AC OPF framework is used, b) the focus is made on LMPs and their formation, c) a new presentation of a marginal resource is contributed, and d) a class of different intertemporal constraints is considered and a methodology for its analysis is proposed. These can be implemented into daily economic dispatch carried out by system and trade operators using intertemporal constraints to optimally allocate available resources and understand locational marginal pricing behind it.

The paper is organized as follows. Section II gives a brief overview of the mathematical background behind LMPs in a system. The proposed methodology of separating out marginal and price-forming resources in the multi-period market is shown in detail in Section III. Section IV is dedicated to LMP formation under intertemporal constraints. The methodology is applied using the IEEE-30 test system, the results of which are shown in Section V. Finally, Section VI draws the conclusions of the paper.

II. Theoretical framework

The general statement of the multi-period market optimal power flow model can be considered as the following programming problem:

$$f = \sum_{t\in\mathcal{T}}\left(\sum_{d\in\mathcal{D}} C_{d,t}P_{d,t} - \sum_{g\in\mathcal{G}} C_{g,t}P_{g,t}\right) \to max, \quad (1)$$

$$g\left(P_{g,t}, P_{d,t}, Q_{d,t}, Q_{d,t}, X_t\right) \le 0, t\in\mathcal{T}, (\lambda) \quad (2)$$

$$h\left(P_{g,t}, P_{d,t}, SC_{s,t}\right) \le 0, t\in\mathcal{T}, (\nu) \quad (3)$$

$$v\left(P_{g,t}, P_{d,t}, Q_{g,t}, SC_{s,t}\right) \le 0, t\in\mathcal{T}, (\pi) \quad (4)$$

The objective of the problem statement (1) is to maximize social benefit across the considered time interval, for example the entire day in a day-ahead market. Constraint (2) represents real and reactive nodal power balances in AC form, transmission and voltage constraints dependent on magnitudes and angles of voltage variables X_t for each time period t. Constraint (3) represents the intertemporal constraints: ramping constraints, energy storage constraints, and energy limited constraints due to fuel limitations, CO_2 emission constraints, and water storage constraints. Constraint (4) represents maximum and minimum levels of the problem's variables.

The Lagrangian function of the stated problem (1)–(4) is the following:

$$L = -f + g^T\lambda + h^T\nu + v^T\pi. \quad (5)$$

To study LMP formation, we need to consider the KKT necessary first-order conditions for derivatives with respect to X, $P_{(g,t)}$, and $P_{(d,t)}$. After they are obtained, we have

$$\frac{\partial L}{\partial P_{g,t}} = C_{g,t} - LMP_{i,t} + J_{h\,i,t}^T\nu + \overline{\pi}_{g,t} - \underline{\pi}_{g,t} = 0, \quad (6)$$

$$\frac{\partial L}{\partial P_{d,t}} = -C_{d,t} + LMP_{i,t} + J_{h\,i,t}^T\nu + \overline{\pi}_{d,t} - \underline{\pi}_{d,t} = 0, \quad (7)$$

$$\frac{\partial L}{\partial X} = J_g^T\lambda = 0, \quad (8)$$

where J_g, J_h are Jacobian matrices of constraints (2) and (3) respectively. Jacobian matrix J_g is a block-diagonal matrix consisting of J_t^{node}, $J_t^{tc}, J_t^{vc}, t\in\mathcal{T}$. Vector λ consists of vectors λ_t^{node}, $\lambda_t^{tc}, \lambda_t^{vc}, t\in\mathcal{T}$.

The formulae (6)–(7) bind LMPs with marginal costs of available resources. That is why they allow defining how LMPs at marginal nodes in the system are formed. It can be seen that formula (8) defines formation of LMPs at different nodes at different time periods as far as it contains LMPs at all nodes and provides interdependence between them and other Lagrange multipliers.

Generators (ergo demands) are traditionally called marginal if their resource's outputs are not bounded and $\overline{\pi}_{g,t(d,t)} = \underline{\pi}_{g,t(d,t)} = 0$. Integration of resources in different time periods connected by intertemporal constraints makes things confusing. Formulae (6)–(7) for the mentioned unbound resources are written as follows:

$$LMP_{i,t} = C_{g,t(d,t)} \pm J_{h\,i,t}^{T} \nu, \tag{9}$$

where $C_{g,t(d,t)}$ is the marginal cost of the corresponding resource, $J_{h\,i,t}^{T} \nu$ is the marginal opportunity cost, that is the cost of choosing other resources for production.

The next sections discuss marginal resources with regard to the intertemporal constraints and LMP formation under their influence. Specific models of intertemporal constraints such as ramping constraints, energy limited constraints, and energy storage constraints are considered.

III. Marginal resources in multi-period market

Marginal pricing is the ground for optimal allocation of generating resources to produce electricity with maximum social benefit. The well-known analysis of locational marginal pricing discloses different marginal generators in the system due to economic similarity based on penalty factors and economic diversity caused by transmission and voltage constraints.

A. Ramping Constraints

We formulate ramping constraints as a limited change in power output by ramp-up and ramp-down rates:

$$-R_g^- \le P_{g,t} - P_{g,t-1} \le R_g^+. \tag{10}$$

Suppose a generator meets ramp-up rate constraints during one time range consisting of $m + 1$ time periods. After that, during another period time range consisting of $n + 1$ periods, it meets ramp-down rate constraints. In this case, relations (6) written for those ranges are the following:

$$\begin{gathered}
LMP_{i,t_1} = C_{g,t_1} - \nu^+_{g,t_1+1} - \underline{\pi}_{g,t_1} \\
LMP_{i,t_1+1} = C_{g,t_1+1} - \nu^+_{g,t_1+2} + \nu^+_{g,t_1+1} \\
\dots \\
LMP_{i,t_1+m-1} = C_{g,t_1+m-1} - \nu^+_{g,t_1+m} + \nu^+_{g,t_1+m-1} \\
LMP_{i,t_1+m} = C_{g,t_1+m} + \nu^+_{g,t_1+m} + \overline{\pi}_{g,t_{1+m}} \\
\dots \\
LMP_{i,t_2} = C_{g,t_2} + \nu^-_{g,t_2+1} + \overline{\pi}_{g,t_2} \\
LMP_{i,t_2+1} = C_{g,t_2+1} + \nu^-_{g,t_2+2} - \nu^-_{g,t_2+1} \\
\dots \\
LMP_{i,t_2+n-1} = C_{g,t_2+n-1} + \nu^-_{g,t_2+n} - \nu^-_{g,t_2+n-1} \\
LMP_{i,t_2+n} = C_{g,t_2+n} - \nu^-_{g,t_2+n} - \underline{\pi}_{g,t_2+n}
\end{gathered} \tag{11}$$

By considering the sum of LMPs at node i of generator g for these periods, we have

$$\begin{gathered}
\sum_{t\in\mathcal{T}_1} LMP_{i,t} = \sum_{t\in\mathcal{T}_1} C_{g,t} - \underline{\pi}_{g,t_1} + \overline{\pi}_{g,t_{1+m}} \\
\sum_{t\in\mathcal{T}_2} LMP_{i,t} = \sum_{t\in\mathcal{T}_2} C_{g,t} + \overline{\pi}_{g,t_2} - \underline{\pi}_{g,t_2+n}
\end{gathered} \tag{12}$$

A marginal generator in its traditional sense defines system price of electricity, which is equal to the generator marginal cost.

If at time periods t_1, t_1+m, t_2, t_2+n the generator does not hit its limits and $\underline{\pi}_{g,t_1} = \overline{\pi}_{g,t_{1+m}} = 0, \overline{\pi}_{g,t_2} = \underline{\pi}_{g,t_2+n} = 0$, we have the equality for mean prices:

$$A\left(LMP_{i,t\in\mathcal{T}_1}\right) = A\left(C_{g,t\in\mathcal{T}_1}\right), A\left(LMP_{i,t\in\mathcal{T}_2}\right) = A\left(C_{g,t\in\mathcal{T}_2}\right) \tag{13}$$

The example of a schedule for this situation is shown in Fig. 1, (a). Such a generator will be considered to be marginal in several time periods because its average LMP corresponds with average marginal cost and its output can be changed under variation in system load.

More frequently ramping constraints are met starting with one of the limits - $\underline{P}_g$ or $\overline{P}_g$. There are no grounds to call such a generator marginal. This resource will not change its output that is bounded by minimum, maximum, and ramping level bounds. Consequently, the generator is considered to be bounded and non-marginal. The example of a non-marginal generator under ramping constraints is shown in Fig. 1, (b). One can see that for the non-marginal generator, it is also necessary to consider several time periods.

Nevertheless, we note that as the generator's output increases, its state changes from minimum to maximum possible level moving from $\underline{P}_g$ or $\overline{P}_g$. At the same time, LMPs in the system increaserise from $LMP_{i,t_1} \le C_{g,t_1}$ to $C_{g,t_1+2} \le LMP_{i,t_1+2}$, i.e. the LMP rises beyond the generator's marginal cost.

B. Energy Limited Constraints.

Consider the following constraints written through the sum of active power outputs for energy constraints:

$$\underline{E}_g \le \sum_{t\in\mathcal{T}} P_{g,t}\Delta t \le \overline{E}_g. \tag{14}$$

They mean the restriction on summary energy output for generator g. Under these constraints, a new form of (9) is as follows:

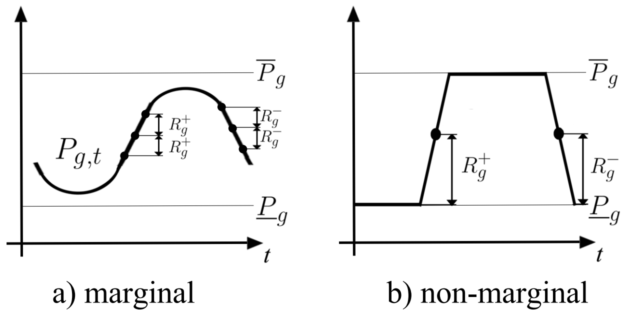

Fig. 1. Generator state under ramping constraints.

$$LMP_{i,t} = C_{g,t} + \overline{\nu}_g^E \Delta t - \underline{\nu}_g^E \Delta t, \quad (15)$$

where $C_{(g,t)} = C_g = idem$.

The marginal opportunity cost for a generator with energy constraints consists of $\overline{\nu}_g^E \Delta t - \underline{\nu}_g^E \Delta t$ and is constant for all considered time periods.

By working with energy constraints of hydro power plants, we can assume that their marginal cost C_g is relatively low in comparison with the prices in the energy system. Regardless of unbounded output at every period, the resource is not marginal. Addressing the bounded maximum energy level constraint for a longer period, we can interpret one whole day as one period. At this integral period, the considered resource is infra-marginal. There is no reason to consider it as marginal during this day.

Another example of energy constraints is a fuel or CO_2 emission-limited power plant with, for example, a relatively high marginal cost. Then, the previous speculations are symmetrical. This resource is extra- marginal (not marginal as well).

The only possible reason to consider such a resource to be marginal is to have non-binding energy constraints with $\underline{\nu}_g^E = \overline{\nu}_g^E = 0$. Then, some time periods will be characterized by power generation at minimum or maximum possible levels. For some time periods, it is necessary for the generator to be unbounded with $LMP_{i,t} = C_{g,t}$. Fig. 2 shows the examples of marginal, infra-marginal and extra-marginal limited energy generator output, respectively.

Thus, through all time periods, e.g. a day, the considered power plant may be infra-marginal, marginal, or extra-marginal, regardless of a single period situation.

C. Energy Storage Constraints

A storage constraint can be introduced by the following expressions:

$$\underline{SC}_s \le SC_{s,t} \le \overline{SC}_s, \quad (16)$$

$$SC_{s,t} = \eta_s SC_{s,t-1} + \eta_d P_{d=s,t}\Delta t - \eta_g P_{g=s,t}\Delta t. \quad (17)$$

Inequalities (16) impose a one-period constraint on minimum and maximum state of charge at time period t while (17) expresses how the state of charge changes in length of time.

Similar to other resources, an energy storage is represented in the target function by the benefit component that is maximized in sum with other components:

$$B_s = \sum_{t\in\mathcal{T}, d=s, g=s} \left(C_{d,t}P_{d,t} - C_{g,t}P_{g,t}\right). \quad (18)$$

It is supposed that $C_{(d,t)} = C_d = idem$, $C_{(g,t)} = C_g = idem$.

This component reveals the arbitrage between periods of low and high LMPs to utilize the storage resource according to its desired marginal cost. As long as a storage benefit component is higher than the social benefit of other resources, it is a signal to charge the storage resource in the low marginal cost periods and to discharge it in the high marginal cost periods.

Under storage constraints, (9) is written as follows:

$$LMP_{i,t} = C_{g(d)} + \eta_{g(d)}\nu_{s,t}\Delta t, \quad (19)$$

where $\nu_{(s,t)}$ are Lagrange multipliers for storage state equality constraints (17). For the analysis of $\nu_{(s,t)}$, we need to consider $\partial L/\partial SC_g$, which is

$$\frac{\partial L}{\partial SC_g} = \nu_{s,t} - \nu_{s,t+1}\eta_s + \overline{\pi}_{s,t}^{SC} - \underline{\pi}_{s,t}^{SC}. \quad (20)$$

With reference to (20), after equating it to zero, we can see that $\nu_{s,t} = \nu_{s,t+1}\,\eta_s$ during charging and discharging of the storage resource. In other time periods, when charge increases to $\overline{SC}_s$ or decreases to $\underline{SC}_s$, the Lagrange multiplier $\nu_{s,t}$ changes its value. Thus, the storage constraints are similar to energy ones during charging and discharging phases only.

The storage resource is marginal if the storage state of charge does not hit $\overline{SC}_s$, $\underline{SC}_s$ during charging phase, i.e., the full storage capacity is not used by the market (Fig. 3).

Taking (17) into consideration, we know that

$$\begin{aligned} B_s = {} & \left(C_d - \frac{\eta_g}{\eta_d}C_g\right)\sum_{t\in\mathcal{T}} P_{d,t} + \\ & \frac{\eta_g}{\eta_d}\frac{C_g}{\Delta t}(1-\eta_s)\sum_{t\in\mathcal{T}\setminus t_{end}} SC_{s,t} + \\ & \frac{\eta_g}{\eta_d}\frac{C_g}{\Delta t}\left(SC_{s,t_{end}} - \eta_s SC_{s,t_{start}}\right). \end{aligned} \quad (21)$$

Considering that the effective energy storage with η_s=1 and without negative LMPs at the current and previous time periods resulted in $SC_{s,t_{end}} = SC_{s,t_{start}} = 0$, we have

$$B_s = \left(C_d - \frac{\eta_g}{\eta_d}C_g\right)\sum_{t\in\mathcal{T}} P_{d,t}. \quad (22)$$

Marginal cost of the storage resource under considered conditions would be $C_d - \frac{\eta_g}{\eta_d}C_g$. It characterizes the LMP

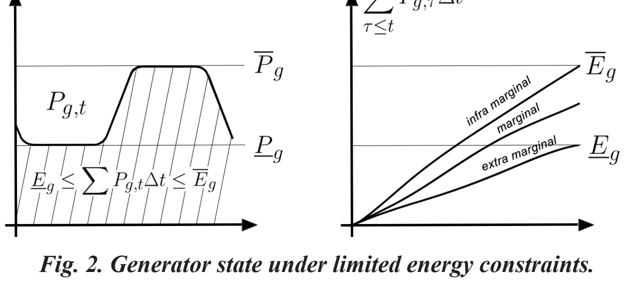

Fig. 2. Generator state under limited energy constraints.

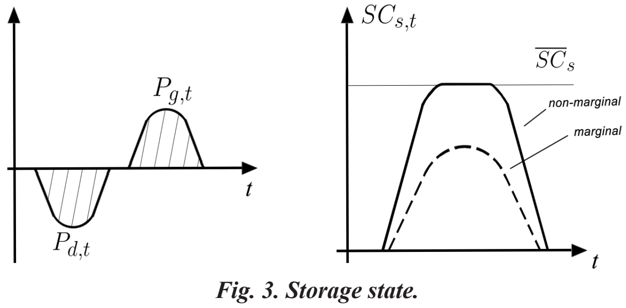

Fig. 3. Storage state.

difference at the storage resource node during charging and discharging time periods, while LMPs at those time periods remain the same. Unlike other types of resources, for which marginal costs are represented in LMPs, the difference of LMPs in the storage resources varies:

$$\Delta LMP_{i,t} = C_g - C_d + \left(\eta_g \nu_{s,t_g} - \eta_d \nu_{s,t_d}\right)\Delta t, \quad (23)$$

where, apart from the efficiency factor η_g,η_d, Lagrange multiplier ν_{s,t_d} increases to ν_{s,t_g}, if Lagrange multipliers $\overline{\pi}^{SC}_{s,t}, \underline{\pi}^{SC}_{s,t}$ corresponding to maximum and minimum levels of state of charge are nonzero. It means that $\Delta LMP_{i,t}$ is greater or equal to price difference $C_g - C_d$ for both marginal and non-marginal storage resources.

Nevertheless, the mutually optimized demand and generation sides of storage just conform to current price situation and do not set prices independently. The marginal storage resource allows reloading conventional resources in order to obtain the desired difference of LMPs. It affects the range but not the size of LMPs.

D. Marginal and Price-forming Resources

As was shown above, the introduction of intertemporal constraints in the multi-period market requires a more formal definition of a marginal resource. Moreover, it is necessary to determine how different types of marginal resources affect prices.

We propose a definition of marginal resource as a resource that is not under constraints (not fully utilized) and can change its output under small changes in the system either at a specific time period or integrally in several time periods. Marginal resources are a) a conventional generator or demand, which is unbounded, b) a generator limited by ramping constraints and unbounded before and after constrained time periods, c) a generator with a non-binding limited energy constraint, and d) a storage resource with non-binding state of charge.

Not all marginal resources form LMPs. We propose a definition of a price-forming resource as a resource whose offer or bid cost directly affects LMPs. Such resources are a)–c) ones mentioned above. Price-forming resources are not e) any resource with binding maximum or minimum output level constraints, f) a generator limited by ramping constraints and bounded before or after constrained time periods, g) a generator with a binding limited energy constraint, and h) a storage resource with fully utilized state of charge.

IV. LMP Formation

Writing (8) in an extensive form for each time period t, we have

$$(J_t^{node})^T \lambda_t^{node} + (J_t^{tc})^T \lambda_t^{tc} + (J_t^{vc})^T \lambda_t^{vc} = 0, \quad (24)$$

where λ_t^{node} consists of real and reactive LMPs at all nodes.

Equation (24) was comprehensively used to define spatial structure of LMP in the one-period market. According to [7], LMPs can be represented by LMP of marginal node through price-bonding factors (PBF) caused by power flow, transmission constraints, and voltage constraints:

$$\begin{aligned}\lambda_t^{pt} &= -\left[\left(J_t^{pt}\right)^T\right]^{-1}\left[\left(J_t^{pf}\right)^T \lambda_t^{pf} + (J_t^{tc})^T \lambda_t^{tc} + (J_t^{vc})^T \lambda_t^{vc}\right] \\ &= \Delta\lambda_t^{f} + \Delta\lambda_t^{tc} + \Delta\lambda_t^{vc}\end{aligned} \quad (25)$$

where λ_t^{pt}, λ_t^{pf} are price-taking and price-forming Lagrange multipliers in λ_t^{node}; J_t^{pt}, J_t^{pf} are corresponding to price-taking and price-forming Jacobian matrices derived in [7], and $\Delta\lambda_t^{f}, \Delta\lambda_t^{tc}, \Delta\lambda_t^{vc}$ are corresponding LMP components due to power flow, transmission and voltage constraints. This is schematically shown in Fig. 4 where resources of type a belong to a set of conventional marginal resources $\mathcal{A}_t$.

The knowledge of marginal price formation is based on the fact that marginal resources will respond to the incremental change in demand at node j. Their output change multiplied by marginal cost will define LMP at the node.

To take advantage of the methodology, it is necessary to determine the difference of $\lambda_t^{pf}, \lambda_t^{tc}, \lambda_t^{vc}$ for the one- and multi-period markets.

For any OPF model, we know that every shadow price of interest can be derived as

$$\lambda^{tc(vc)} = \partial f / \partial Lm^{tc(vc)}$$

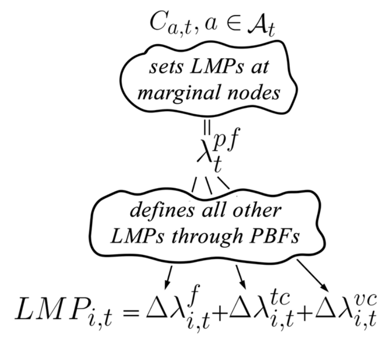

Fig. 4. Scheme of LMP formation without influence of intertemporal constraints.

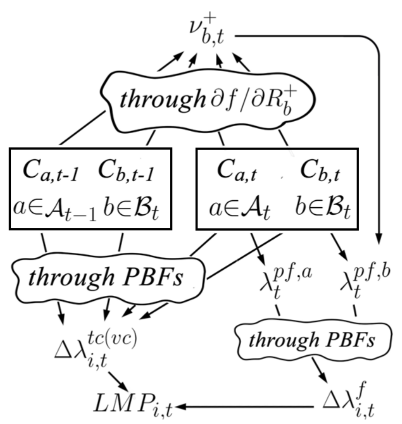

Fig. 5. Temporal scheme of LMP formation with influence of ramping rates.

$$\sum_{t\in\mathcal{T}}\left(\sum_{d\in\mathcal{D}} C_{d,t}\frac{\partial P_{d,t}}{\partial Lm^{tc(vc)}}-\sum_{g\in\mathcal{G}} C_{g,t}\frac{\partial P_{g,t}}{\partial Lm^{tc(vc)}}\right), \quad (26)$$

where Lm is the corresponding limit (maximum power flow, minimum or maximum voltage magnitude).

Unlike the one-period market, in the multi-period market the sum over time periods in (26) reveals the response of variable output of marginal and other resources in several time periods connected by intertemporal constraints. Thus, the transmission and voltage constraints components in LMPs are linked through PBF to different resources from different time periods.

As for price-forming LMPs λ_t^{pf}, furthermore, we consider different marginal and non-marginal resources with variable output and define price-taking LMP at their nodes under intertemporal constraints.

A. Influence of Ramping Rates

According to (11), LMP at a marginal generator under ramping constraints at a specific time period is diverted from a marginal cost. The reason is the use of other resources to meet the constraints. Knowing that

$$\nu_{g,t}^{\pm}=\frac{\partial f}{\partial R_g^{\pm}}=\sum_{t\in\mathcal{T}}\left(\sum_{d\in\mathcal{D}} C_{d,t}\frac{\partial P_{d,t}}{\partial R_g^{\pm}}-\sum_{g\in\mathcal{G}} C_{g,t}\frac{\partial P_{g,t}}{\partial R_g^{\pm}}\right) \quad (27)$$

$\lambda_{i,t}^{pf}$ at a corresponding marginal node can be represented as follows

$$\lambda_{i,t}^{pf}=C_{g,t}\pm\nu_{g,t}^{\pm}=\sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}} k_{r,t}^{R_g^{\pm}} C_{r,t}\,, \quad (28)$$

where $k_{r,t}^{R_g^{\pm}}$ are the factors received from (27) plus one for the generator under consideration.

It is worth noting that partial derivatives $\partial P_{g,t(d,t)}/\partial R_g^{\pm}$ in the previous equation are nonzero only for marginal resources. For example $\sum_{t\in\mathcal{T}}\partial P_{g,t}/\partial R_g^{\pm}$ for non-marginal limited energy constraints will be equal to zero.

As a result, marginal LMP at the node of the considered generator is equal to its marginal cost plus marginal opportunity cost, namely, weighted costs of other marginal resources from the current and adjacent hours.

Substituting $\lambda_{i,t}^{pf}$ into (25), we find a new version of LMP component due to power flow. Note that the sensitivities obtained in (26) already take into consideration binding ramping constraints. The overall scheme of ramping rates influence for the time period t is shown in Fig. 5. Here the sets of marginal generators $\mathcal{A}_t, \mathcal{A}_{t-1}$ belong to conventional marginal generators of type a, while the set of marginal generators $\mathcal{B}_t$ refers to marginal generators of type *b* with binding ramping constraints. The similar scheme can be drawn for the time period *t*-1.

B. Influence of Limited Energy Constraints

As was said above, marginal generators with non-binding energy constraints are similar to conventional generators. Non-marginal limited energy generators are of more interest. They allow rescheduling generators in order to smooth out load variation. As a consequence, according to (15), such an ability of generator to shift its output to another time period with different hourly binding transmission, voltage, and other constraints allows us to form the same LMP at the limited energy generator node.

Rescheduling leads to selecting marginal resources with similar marginal costs during different time periods. It leads to the conclusion that LMP at the node of the limited energy generator comprises marginal costs at all time periods. PBFs for the LMP can be calculated for each time period and considered with weight of one divided by the number of such periods:

$$LMP_i=\sum_{r\in\mathcal{R}}\sum_{c=\{f,tc,vc\}} A\left(k_{r,t\in\mathcal{T}}^{c}\right)C_r\,, \quad (29)$$

where C_r here unifies same offer or bid prices for a resource r throughout the considered time period.

We can also consider a specific time period when there are no price-forming resources are present. For example, all generators with relatively low costs reach their maximum levels and other generators with relatively high costs are not utilized yet. The price in this case will be set by marginal generators from other time periods and transferred through the limited energy generator. The same refers to all time periods although the price-forming property is not apparent for the general case.

Thus, on the one hand, the limited energy resource cost is an opportunity marginal cost formed by actual marginal resources from all time periods, on the other hand, the limited energy resource transfers the received cost as an LMP at all specific time periods.

C. Influence of Energy Storage Constraints

The influence of the energy storage constraints on LMPs is similar to the influence of limited energy constraints yet with some differences. The first one is in two energy limited periods — charging and discharging periods. The second difference is that marginal or non-marginal storage resources reschedule conventional generators to provide LMP difference to be greater than or equal to its desired marginal cost. We recall that a marginal limited energy generator ceases to connect different hours while storage

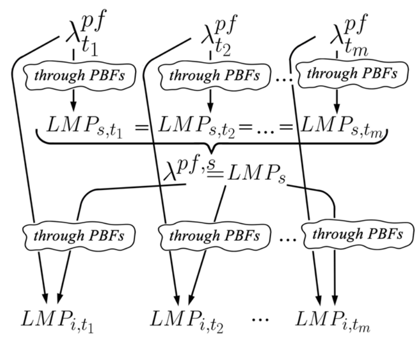

Fig. 6. Temporal scheme of LMP formation with influence of limited energy and energy storage constraints

continues to unite hoursdoing that independently of its status. Thus, all the above considerations on limited energy constraints can be applied to energy storage. In the overall scheme of limited energy constraints influence shown in Fig. 6, s denotes a limited energy generator, a storage generator, or a storage demand.

All the above considerations are summarized in Table I.

V. Illustrative examples

The proposed approach is demonstrated on the IEEE-30 power system with demands following a daily profile shown in Fig. 7. AC OPF was run in 24-hour periods from 0 to 23 hours usual for day-ahead markets.

All input and output data, as well as an exact mathematical model of the OPF problem and enumerated modifications in the test energy system, can be found in [33].

Table 1. LMP formation at marginal nodes

Resource	Status	Conditions to be met
Conventional	Extra marginal	$P_{g,t(d,t)} = \underline{P}_{g(d)}$, $LMP_{i,t} \le C_{g,t}$ or $LMP_{i,t} \ge C_{d,t}$
	Marginal	$\underline{P}_{g(d)} < P_{g,t(d,t)} < \overline{P}_{g(d)}$, $LMP_{i,t} = C_{g,t(d,t)}$
	Infra marginal	$P_{g,t(d,t)} = \overline{P}_{g(d)}$, $LMP_{i,t} \ge C_{g,t}$ or $LMP_{i,t} \le C_{d,t}$
Under ramping constraints	Extra marginal	$\underline{P}_g < P_{g,t} < \overline{P}_g$, $A(LMP_{i,t}) \le A(C_{g,t})$
	Marginal	$\underline{P}_g < P_{g,t} < \overline{P}_g$, $A(LMP_{i,t}) = A(C_{g,t})$
	Infra-marginal	$\underline{P}_g < P_{g,t} < \overline{P}_g$, $A(LMP_{i,t}) \ge A(C_{g,t})$
Energy limited	Extra-marginal	$\sum_{t\in\mathcal{T}} P_{g,t}\Delta t = \underline{E}_g$, $LMP_{i,t} = \sum_r \sum_c A(k^c_{r,t\in\mathcal{T}^*})C_r$, $\mathcal{T}^* = \{t: \underline{P}_g < P_{g,t} < \overline{P}_g\}$
	Marginal	Same as conventional[1]
	Infra-marginal	$\sum_{t\in\mathcal{T}} P_{g,t}\Delta t = \overline{E}_g$ $LMP_{i,t}$ is the same as for extra-marginal energy limited resource
Storage	Extra-marginal	$SC_{s,t} = \underline{SC}_s$
	Marginal	$\underline{SC}_s < SC_{s,t} < \overline{SC}_s$ For charging phase: $LMP_{i,t} = \sum_r \sum_c A(k^c_{r,t\in\mathcal{T}^{ch}})C_r$, $\mathcal{T}^{ch} = \{t: \underline{P}_d < P_{d,t} < \overline{P}_d\}$ For charging phase: $LMP_{i,t} = \sum_r \sum_c A(k^c_{r,t\in\mathcal{T}^{dch}})C_r$, $\mathcal{T}^{dch} = \{t: \underline{P}_g < P_{g,t} < \overline{P}_g\}$
	Infra-marginal	$SC_{s,t} = \overline{SC}_s$, $LMP_{i,t}$ is the same as for marginal storage resource

[1] Applicable for extra-marginal, marginal, and infra-marginal cases which are met during the considered time period

We will examine three cases. The first one shows LMP formation under the influence of ramping rates. The second case evaluates the change in LMP after placement of a limited energy generator in the system. The third case, on the contrary, assumes the introduction of a storage resource additionally to the first case.

A. Case 1: LMPs with Ramping Rates

For each of the six generators in the 30-node system we formulate several offers with different price levels and a capacity greater or equal to 10 MWh. At the same time, we set ramping rates to 5 MWh so that none of generators can freely increase or decrease its output. To introduce a diverse spatial LMP structure, we set a maximum active flow through line 6–8 to 25.4 MW. The results of the economic dispatch (1)–(4), specifically generators' schedule and LMPs, are shown in Figs. 8 and 9.

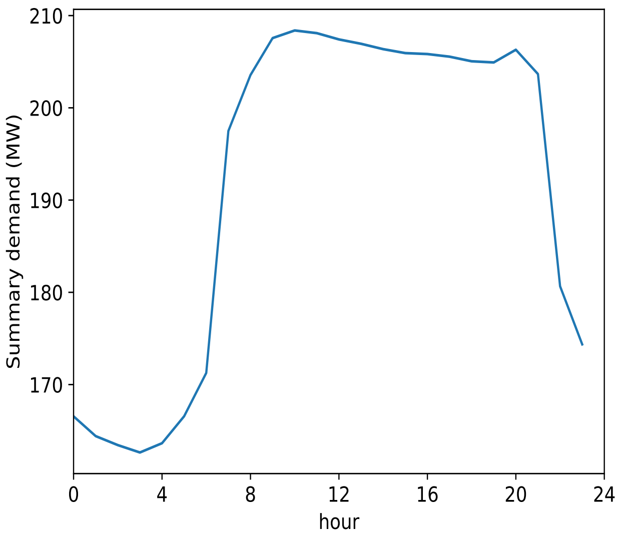

Fig. 7. Load hourly profile.

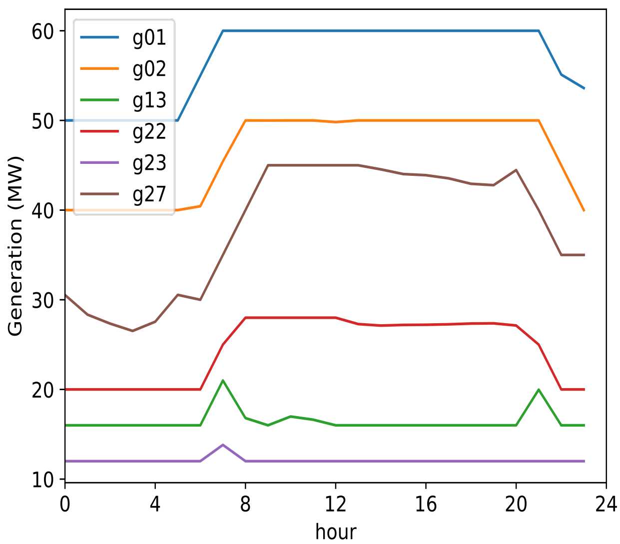

Fig. 8. Generator power output under ramping constraints in case 1.

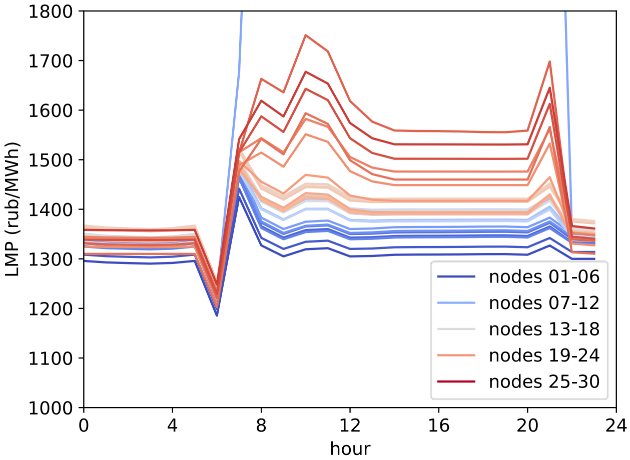

Fig. 9. LMPs under ramping constraints in case 1.

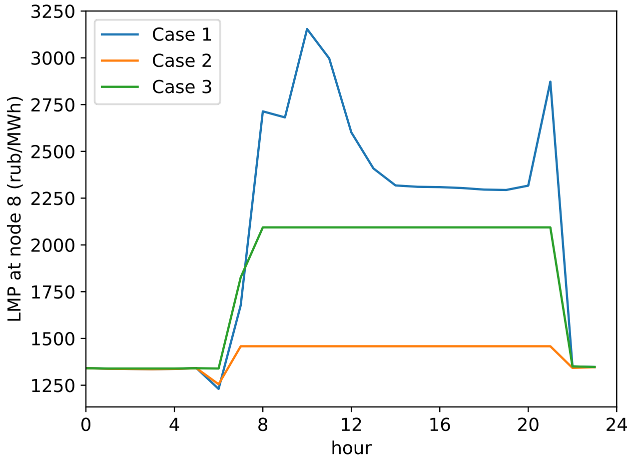

Fig. 10. LMP at node 8 in different cases.

Ramping constraints are hit in this case by generators 1, 2, 13, and 22 at hour 7 with ramp-up rate; by generator 27 at hours 7 and 8 with ramp-up rate; by generators 22 and 27 at hour 22 with ramp-down rate; by generator 2 at hours 22–23 with ramp-down rate. Only generator 2 became marginal under ramping constraints at hours 6–7. The other conventional marginal generators can be found in Table II.

LMPs at node 2 at hours 6–7 are 1198.39 and 1441.61 rub/MWh, respectively. Notably, average LMP is strictly equal to 1320 rub/MWh, which is marginal cost of generator 2. According to (11) and (28), and having $v^{+}_{2,7} = 0.974062\ C_{23,7} - C_{2,6} = 121.61$ rub/MWh LMPs at node 2 at hours 6–7 are formed in the following way:

$$\begin{aligned} LMP_{2,6} &= C_{2,6} - v^{+}_{2,7} = 2C_{2,6} - 0.974062C_{23,7} \\ LMP_{2,7} &= C_{2,7} + v^{+}_{2,7} = 0.974062C_{23,7} \end{aligned} \quad (30)$$

As to the other generators under ramping constraints, the LMPs at node of generator 1 are 1185.44 and 1424.56 rub/MWh, respectively. Average LMP is 1305.00 rub/MWh and it is higher than the marginal cost of 1300 rub/MWh.

Thus, generator 1 is infra-marginal. On the contrary, generator 13's LMPs are 1223.65 and 1477.04 rub/MWh, respectively, while its marginal costs are 1400 and 1450 rub/MWh, respectively. An average LMP of 1350.35 rub/MWh versus an average marginal cost of 1425 rub/MWh makes this generator extra-marginal.

Note that no generator actually has marginal costs below 1300 rub/MWh and that LMPs at hour 6 are formed at lower level by the only marginal generator under ramping constraints with the LMP of 1198.39 rub/MWh at marginal node. It is formed by two marginal costs of generator 2 at hour 6 and generator 23 at hour 7.

Note also that at hour 7, generator 23 is marginal at two nodes: 23 with a marginal cost of 1480 rub/MWh and 2 with a marginal cost of 0.974062·1480 rub/MWh. However, the set of replacing resources in general can vary.

All generators under ramping constraints need help to follow the demand, so the output of the most expensive offer of generator 23 (1480 rub/MWh) was raised at hour 7. At the adjacent hour 6, the cheapest offer of generator 27 (1310 rub/MWh) was decreased as far as it was allowed by ramp-up rates for its own output at 6–8 hours. Similar help is observed for falling load slope. Generator 13 (1400 rub/MWh) was chosen to help other generators 2, 22, and 27 to meet their ramping constraints.

As is seen from lower blue lines in Fig. 9, LMPs with ramping constraints form "cardiogram" curves. According to (11), the first period is characterized by LMP fall and the last period is characterized by LMP rise. We can also discern "cardiogram" curves for red lines at hours 7–8 where generator 27 met ramp-up rate. While respecting ramp-down rates we observe the opposite effect of an initial increase and then a decrease in the marginal LMPs.

Let us consider the spatial structure of LMP. The binding transmission constraint in line 6–8 divides the system into two parts at hours 7–21. Node 8's LMP has the highest positive transmission component — 860 rub/MWh and above. It is of interest to observe how it changes in different cases, which is shown in Fig. 10.

Other positive components due to the transmission constraint for nodes (red lines in Fig. 9) do not exceed 400 rub/MWh. In this example, transmission constraints act independently of intertemporal constraints despite the ramping connection inside of 6–8 hours. Transmission components are formed by hourly marginal costs as it

Table 2. Marginal generators under ramping constraints in case 1.

Generator	Hours	Marginal Cost	Type
g01	22–23	1300	a) conventional
g02	6–7	1320	b) ramping
	9, 12	1320	a) conventional
g13	8, 10–11, 21	1400	a) conventional
g22	13–20	1390	a) conventional
g23	7	1480	a) conventional
g27	0–5	1310	a) conventional
	14–20	1460	a) conventional

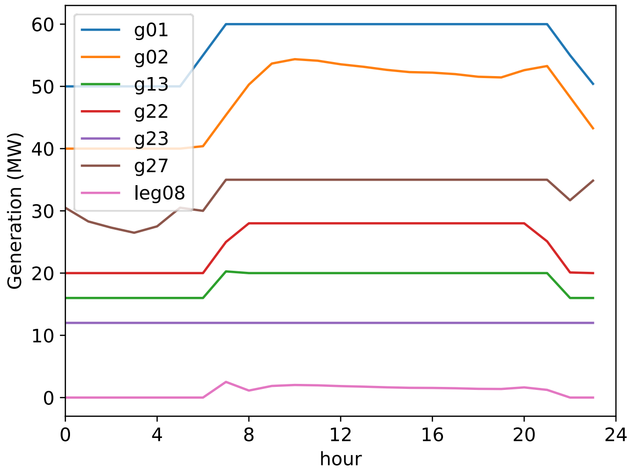

Fig. 11. Generator's power output under ramping constraints with limited energy generator (leg) in case 2.

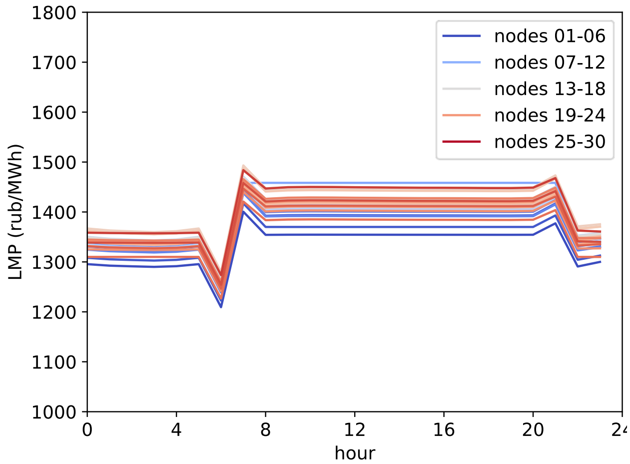

Fig. 12. LMPs under ramping constraints with limited energy generator in case 2.

is studied in previous research. There are also voltage components in a range of -2.12 to 0.03 rub/MWh. Due to small values, there is no need to rigorously study them.

B. Case 2: LMPs with Limited Energy Constraints

To further consider LMP at node 8, a limited energy generator with energy limit of 25 MWh was installed there. This generator can dispatch active power of less than or equal to 10 MW with a comparably low marginal cost. The generator provides the range of reactive power from –5 to 5 MVAr. LMPs and generators' output are shown in Figs. 11 and 12 while marginal generators are given in Table III.

The Figures show that the generators' output and LMPs are smoothed out during 8-20 hours. With the help of the new limited energy generator, the system is able to refuse high price offers of generators 13, 23, and 27 at hours 7–21, when the new generator is fully utilized replacing other resources.

As expected, there are less binding ramping constraints in the system. Ramp-up rates are achieved by generators 1, 2, 22, and 27 at hour 7. Ramp-down rates are reached by generators 1 and 22 at hour 22 and by generator 2 at hour 22–23. Nevertheless, the problem has not been solved. We can see from LMPs in Fig. 12 that the amplitude of the ramping "cardiogram" curve has remained the same. On the other hand, LMP at node 8 was lowered considerably

Table 3. Marginal generators under ramping constraints with limited energy generator in case 2.

Generator	Hours	Marginal Cost	Type
g01	23	1300	a) conventional
g02	6–7, 21–23	1320, 1370–1320	b) ramping
	8–20	1370	a) conventional
g13	7	1450	a) conventional
g22	21–22	1390	b) ramping
g27	0–5, 22–23	1310	a) conventional

(see Fig. 10), but the transmission constraint in line 6–8 is still binding.

The number of marginal generators under ramping constraints has increased. For example, generator 2 has become marginal with ramping type b during a 3-hour interval from hour 21 to hour 23. LMPs at node 2 at those hours are 1393.05, 1304.33, and 1312.62 rub/MWh, respectively. Offer prices are 1370, 1320, and 1320 rub/MWh, respectively. An average value of both price arrays is the same and is equal to 1336.66 rub/MWh. LMPs at the price-forming node 2 are formed taking into account the opportunity cost:

$$LMP_{2,21} = C_{2,21} + \nu^{-}_{2,22} = 1393.05,$$
$$LMP_{2,22} = C_{2,22} - \nu^{-}_{2,22} + \nu^{-}_{2,23} = 1304.33,$$
$$LMP_{2,23} = C_{2,23} - \nu^{-}_{2,23} = 1312.62,$$
$$\nu^{-}_{2,22} = 0.803762\left(C_{2,22} + C_{2,23}\right) - 0.196238C_{2,21} +0.195997\left(C_{22,21} + C_{22,22}\right) - 1.001186C_{27,22} -0.689971C_{1,23} - 0.120668C_{27,23} -0.005732C_{13,7} = 23.05,$$
$$\nu^{-}_{2,23} = 0.887871C_{2,22} - 0.112129\left(C_{2,21} + C_{2,22}\right) +0.112075\left(C_{22,21} + C_{22,22}\right) - 0.762173C_{1,23} -0.133295C_{27,23} - 0.003237C_{27,22} -0.003358C_{13,7} = 7.38.$$

Marginal cost $C_{13,7}$ (1450 rub/MWh) is connected with hours 21–23 through the binding energy limited constraint at hours 7–21. We can explain it by the following chain of events. The output of generator 13 at hour 7 should be slightly increased to lower the output of limited energy generator 8 at the same hour in order to support the ramping constraints of generator 2 at hours 22-23 by increasing its output at hour 21.

Let us consider the LMP formation at node 8 and its influence on price-taking LMPs during hours 7–21. The LMP is equal to 1458.33 rub/MWh. Price-forming resources make the following LMP components: $\Delta\lambda^{f}_{8,7-21} = 1418.05$, $\Delta\lambda^{tc}_{8,7-21} = 40.28$. Each of them is

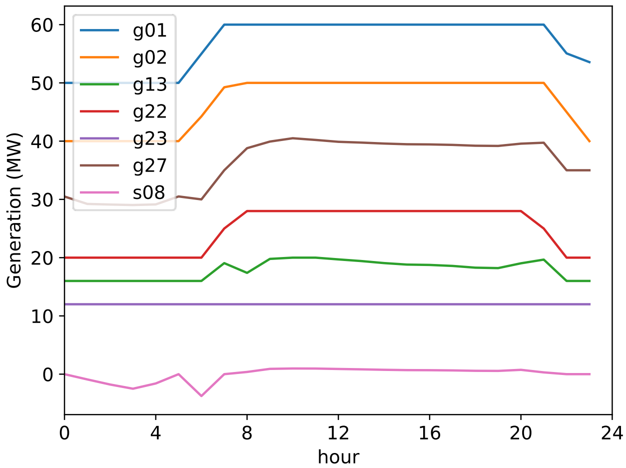

Fig. 13. Generator's power output under ramping constraints with storage (s) in case 3.

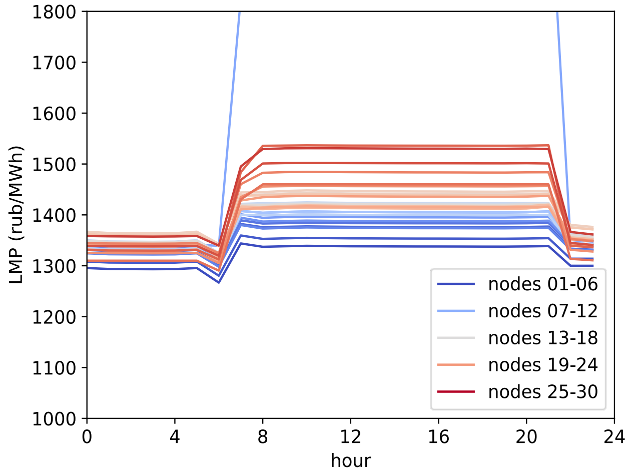

Fig. 14. LMPs under ramping constraints with storage in case 3.

connected with price-forming marginal costs through price-bonding factors. After determining the limited energy generator LMP at considered hours, we can make the next step and switch the status of the node to marginal. Its variable output is responsive to changes inside each time period. Node 8, being price-forming, makes spatial LMP structure at hours 8–20 clear. There are two price-forming nodes – each for zones of high and low LMPs under the influence of transmission congestion in line 6–8. Conventional marginal generator 2 supports incremental changes in demands and determines LMPs at nodes 1–7, 9–12 (blue lines in Fig. 12). An incremental change in demand for nodes 19–30 (red lines in Fig. 12) is handled by node 8.

C. Case 3: LMPs with Energy Storage

In case 3, we choose a storage resource to be installed at node 8 instead of the limited energy generator. The storage resource is defined by the following parameters: $\overline{SC}_s = 10$ MWh, $\eta_g = 1.01$, $\eta_d = 0.95$, $C_d = 0$, $C_g = 500$ rub/MWh. Optimization results after replacing the resource are shown in Figs. 13 and 14. Marginal generators are listed in Table IV.

The results show that the storage resource has a more significant effect on LMPs in comparison to a limited energy generator. The reason lies in the demand side. At hour 6, the demand of the storage resource has replaced increased output of generators 13 and 27 at hour 7 in case 1. This considerably reduced the effect of ramping constraints on LMPs. Nevertheless, as is seen from Fig. 10, the LMP at node 8 has not changed much because of a required marginal cost of the storage resource and its inefficiency.

Charging phase of the storage resource begins at hour 1 and continues until hour 6, excluding hour 5. During this phase, LMP is equal to 1339.37 rub/MWh and consists of $\Delta\lambda^{f}_{8,t_{ch}} = 1334.98$ and $\Delta\lambda^{tc}_{8,t_{ch}} = 4.39$. Discharging phase lasts from hour 8 to hour 21 with LMP formed to be 2093.75 rub/MWh. This LMP is composed of $\Delta\lambda^{f}_{8,t_{dch}} = 1464.33$ and $\Delta\lambda^{tc}_{8,t_{dch}} = 629.43$. Both LMPs are price-forming inside specific time periods.

Thus, they augment the list of marginal generators in Table IV with costs 1339.37 and 2093.75 rub/MWh, which in turn were comprised by given marginal costs of all connected hours.

Table 4 marginal generators under ramping constraints with storage in case 3

Generator	Hours	Marginal Cost	Type
g01	22–23	1300	a) conventional
g02	6–7	1320	b) ramping
g13	7–9, 12–21	1400	a) conventional
g27	0–5	1310	a) conventional
	8–21	1460	a) conventional

VI. Conclusion

This paper proposes a new methodology to express Locational Marginal Prices (LMPs) as the sum of spatial components due to transmission and voltage constraints, and temporal components due to intertemporal constraints. The most common forms of intertemporal constraints are taken into consideration, namely ramping constraints, limited energy constraints, and storage constraints. The proposed approach is innovative in introducing a new definition of multi-period marginal and price-forming resources and a novel technique to uncover the dependence of the LMPs on various types of marginal resources from different time periods.

LMP decomposition is done for opportunity costs of marginal resources under intertemporal constraints. Each such resource brings marginal costs of adjacent periods multiplied by price-bonding factors into a current LMP structure. However, it is shown that the influence of the intertemporal constraints on the LMP varies considerably. Ramping constraints lead to "cardiogram" LMP curves. Limited energy and storage constraints smooth out the LMPs and price-bonding factors (PBFs) throughout the considered period. Although the marginal status of storage refers rather to LMP difference during high and low pricing periods.

This paper handles the multi-period AC OPF in order to calculate the LMPs, which helps to reflect intertemporal constraints in the system when determining the LMPs. The developed methodology provides a complex temporal structure of price signals that can support useful information about the profitability of placing additional resources to manage net load variability and system congestion. The methodology has been tested on a 30-node energy system.

References

[1] W. W. Hogan, "Contract networks for electric power transmission," J. Regul. *Econ.*, vol. 4, no. 3, pp. 211–242, 1992.

[2] W. W. Hogan, E. G. Read, and B. J. Ring, "Using mathematical programming for electricity spot pricing," *Int. Trans. Oper. Res.*, vol. 3, no. 3–4, pp. 209–221, 1996.

[3] M. Caramanis, R. Bohn, and F. C. Schweppe, "Optimal spot pricing: practice and theory," IEEE Trans. Power Appar. Syst., vol. PAS-101, no. 9, pp. 3234–3245, Sep. 1982.

[4] R. E. Bohn, M. C. Caramanis, and F. C. Schweppe, "Optimal pricing in electrical networks over space and time," *Rand J. Econ*., vol. 15, no. 3, pp. 360–376, Sep. 1984.

[5] F. Schweppe, M. Caramanis, R. Tabors, and R. Bohn, "Spot pricing of electricity. Boston: Kluwer", *Kluwer Academic Publishers*, 1988.

[6] T. Orfanogianni and G. Gross, "A General Formulation for LMP Evaluation," *IEEE Trans. Power Syst.*, vol. 22, no. 3, pp. 1163–1173, Aug. 2007.

[7] T. Vaskovskaya, P. Guha Thakurta, and J. Bialek, "Contribution of transmission and voltage constraints to the formation of locational marginal prices," *Int. J. Electr. Power Energy Syst.*, vol. 101, pp. 491–499, Oct. 2018.

[8] V. K. Singh and S. K. Singal, "Operation of hydro power plants-a review," *Renew. Sustain. Energy Rev.*, vol. 69, pp. 610–619, Mar. 2017.

[9] A. J. Wood, B. F. Wollenberg, and G. B. Sheble, "Power generation, operation, and control", 3rd ed., Wiley, 2013.

[10] M. T. L. Barros, F. T.-C. Tsai, S. Yang, J. E. G. Lopes, and W. W.-G. Yeh, "Optimization of large-scale hydropower system operations," J. Water Resour. Plan. Manag., vol. 129, no. 3, pp. 178–188, May 2003.

[11] O. Hoseynpour, B. Mohammadi-ivatloo, M. Nazari-Heris, and S. Asadi, "Application of dynamic non-linear programming technique to non-convex short-term hydrothermal scheduling problem," *Energies*, vol. 10, no. 9, pp. 1–17, Sep. 2017.

[12] C. P. Nguyen, M. Marwali, M. Swider, C. Rosecrans, and C. Sanada, "Limited energy resource optimization in the electricity market," in *Proc. IEEE Power and Energy Society General Meeting*, 2018, pp. 1–5.

[13] E. Barbour, I. A. G. Wilson, J. Radcliffe, Y. Ding, and Y. Li, "A review of pumped hydro energy storage development in significant international electricity markets," *Renew. Sustain. Energy Rev.*, vol. 61, pp. 421–432, Aug. 2016.

[14] "Electric storage participation in markets operated by regional transmission organizations and independent system operators," *Federal Energy Regulatory Commission, Fed. Regist.*, vol. 83, no. 44, pp. 9580–9633, 2018.

[15] A. Sakti, A. Botterud, and F. O'Sullivan, "Review of wholesale markets and regulations for advanced energy storage services in the United States: Current status and path forward," *Energy Policy*, vol. 120, pp. 569–579, 2018.

[16] J. Zhao, T. Zheng, and E. Litvinov, "A Multi-Period Market Design for Markets with Intertemporal Constraints," arXiv e-prints, pp. 1–12, Dec. 2018. [Online]. Available: http://arxiv.org/abs/1812.07034.

[17] S. Zaferanlouei, M. Korpas, J. Aghaei, H. Farahmand, and N. Hashemipour, "Computational efficiency assessment of multi-period AC optimal power flow including energy storage systems," in Proc. 2018 *International Conference on Smart Energy Systems and Technologies, SEST* 2018 - Proceedings, 2018, pp. 1–6.

[18] J. A. Taylor, "Financial storage rights," *IEEE Trans. Power Syst.*, vol. 30, no. 2, pp. 997–1005, 2015.

[19] D. Muñoz-Álvarez and E. Bitar, "Financial storage rights in electric power networks," *J. Regul. Econ., vol.* 52, no. 1, pp. 1–23, Aug. 2017.

[20] V. Popović, B. Jereb, M. Kilibarda, M. Andrejić, A. Keshavarzsaleh, and D. Dragan, "Electric vehicles as electricity storages in electric power systems," *Logist. Sustain. Transp.*, vol. 9, no. 2, pp. 57–72, Oct. 2018.

[21] A. Khodabakhsh, O. Papadigenopoulos, J. Matuschke, J. Horn, E. Nikolova, and E. Pountourakis, "Electric Vehicle Valet," arXiv e-prints, pp. 1–8, Nov. 2018. [Online]. Available: http://arxiv.org/abs/1811.06184.

[22] K. Bradbury, L. Pratson, and D. Patiño-Echeverri, "Economic viability of energy storage systems based on price arbitrage potential in real-time U.S. electricity markets," *Appl. Energy,* vol. 114, pp. 512–519, Feb. 2014.

[23] L. Yao, B. Yang, H. Cui, J. Zhuang, J. Ye, and J. Xue, "Challenges and progresses of energy storage technology and its application in power systems," *J. Mod. Power Syst. Clean Energy*, vol. 4, no. 4, pp.

519–528, Oct. 2016.

[24] C. K. Das, O. Bass, G. Kothapalli, T. S. Mahmoud, and D. Habibi, “Overview of energy storage systems in distribution networks: Placement, sizing, operation, and power quality,” *Renew. Sustain. Energy Rev*., vol. 91, pp. 1205–1230, Aug. 2018.

[25] P. Denholm, M. O’Connell, G. Brinkman, and J. Jorgenson, “Overgeneration from Solar Energy in California. A Field Guide to the Duck Chart,” *Natl. Renewable Energy Lab., Tech. Rep*. NREL/TP-6A20–65023, Nov. 2015.

[26] “What the Duck Curve Tells Us About Managing a Green Grid,” *California ISO, Shap. a Renewed Futur.*, pp. 1-4, 2015.

[27] B. Hua, D. A. Schiro, T. Zheng, R. Baldick, and E. Litvinov, “Pricing in multi-interval real-time markets,” *IEEE Trans. Power Syst.*, vol. 34, no. 4, pp. 2696–2705, Jul. 2019.

[28] P. Zou et al., “Evaluating the impacts of flexible ramping products on the market equilibrium, in *Proc. IEEE Power Energy Soc. Gen. Meet*., 2016, pp. 1–5.

[29] A. Cornelius, R. Bandyopadhyay, and D. Patiño-Echeverri, “Assessing environmental, economic, and reliability impacts of flexible ramp products in MISO’s electricity market,” *Renew. Sustain. Energy Rev.*, vol. 81, no. 2, pp. 2291–2298, Jan. 2018.

[30] M. R. Davidson et al., “Mathematical model of power system management in conditions of a competitive wholesale electric power (capacity) market in Russia,” *J. Comput. Syst. Sci. Int.*, vol. 48, no. 2, pp. 243–253, Apr. 2009.

[31] “General Information About Zagorskaya Hydtor Power Plant,” RusHydro website. [Online]. Available: http://www.zagaes.rushydro.ru/pshpp/general/ (visited on 05/30/2019).

[32] E. Ela and M. O’Malley, “Scheduling and Pricing for Expected Ramp Capability in Real-Time Power Markets,” *IEEE Trans. Power Syst*., vol. 31, no. 3, pp. 1681–1691, May 2016.

[33] T. Vaskovskaya. “IEEE-30 energy system data of multi-period market with intertemporal constraints,” *Zenodo Repository,* DOI: 10.5281/zenodo.2702758, 2019.

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