General Methodological Approaches to Hierarchical Modeling of Complex Systems

N. I. Voropai[*]

Melentiev Energy Systems Institute of Siberian Branch of Russian Academy of Sciences, Irkutsk, Russia

***Abstract* — Based on the analysis of features inherent in any complex system, including modern energy systems, this study argues for the necessity of adopting hierarchical principles of modeling of such systems when solving the tasks of substantiating their development and the way their operation is controlled. The study also provides a concise overview of general methodological research contributions that further the development of the technology behind hierarchical modeling of complex systems.**

***Index Terms* — Complex systems, hierarchical modeling, general methodological approaches.**

I. INTRODUCTION

By the second half of the 20th century, academics and decision makers alike faced the need to handle complex large-scale systems of various nature in the process of researching them, and most importantly when substantiating their development and the way the operation of such systems is controlled. Naturally, this situation did not arise out of the blue, but, metaphorically speaking, was "ripening" at a steady pace as man-made systems were developing and growing in complexity along with our becoming aware of the need to treat the real world we live in from the systems viewpoint. At the same time, defining tenets and the very structure of the theory of large systems were articulated, based on the fundamental principle that stipulates that any theory that claims to study complex systems should do so by operating the models, the structure of which reflects this complexity [1 - 3, etc.]. Treating energy systems as complex large systems was typical of this period with respect to the energy sector problems as well [4 - 6, etc.].

It is self-explanatory that the solution to the majority of problems of substantiating the development and operation control when dealing with large systems if undertaken in the "head-on" fashion proves to be the source of substantial methodological challenges owing to bulkiness and immensity of multivariance and uncertainty of external conditions, the availability of multiple decision criteria, etc. which are all prerequisites for using such models. Herein we omit from our consideration some problems that can be handled reasonably well by means of relatively simple models of a given system. The complexity of solving problems on the basis of using complex models makes us decompose the initial problem into its sub-problems, or, in general, into a hierarchy of sub-problems, for the description of which the hierarchy of corresponding models is required. The hierarchy of sub-problems corresponds to the hierarchical organizational structure, each of the elements of which implements its own solution, obtained as a result of solving the corresponding sub-problem.

Consequently, the hierarchical approach is determined not by the complex system under study, the hierarchical structure of which, as a rule, is does not manifest itself clearly, but by the hierarchy of sub-problems to be solved and models employed for this purpose. In other words, the hierarchical representation of a given large system follows from the hierarchy of the sub-problems to be solved as stipulated by the decision maker. It needs no further clarification that the hierarchy of research sub-problems is to a certain extent subjective.

In what follows this study presents a concise overview of general methodological approaches to hierarchical modeling of complex systems.

II. OVERVIEW OF GENERAL METHODOLOGICAL APPROACHES

In general, we are dealing with three hierarchies: the hierarchy of the object of the study: a large system represented by the corresponding hierarchy of models; the hierarchy of problems and solutions based on the former; the hierarchy of the organizational structure that

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[*] Corresponding author.
E-mail: voropai@isem.irk.ru

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implements the solutions obtained. A unique and one-of-a-kind book by M. D. Mesarovic, D. Macko and Y. Takahara dwells at great depth on this cornerstone idea [1]. Accordingly, the authors introduce different terms to distinguish between the above three hierarchies: strata, layers and echelons. The relationship between these three categories of hierarchy is elucidaded as stratified, multi-layered, and multiechelon hierarchies.

Within the framework of the mathematical theory of systems the book presents various concepts of the hierarchy. Concepts of systems, subsystems, and their interrelationships are introduced by means of the set theory apparatus. The book substantiates the importance of formalization of multilevel hierarchical systems, which gives the opportunity to achieve the required accuracy of description, apply mathematical methods, and conduct necessary research. A formalization of the paramount problem of the theory advocated by the authors, that of coordination of elements of the hierarchical structure, is proposed. The formalization that they introduce enables the application of mathematical analysis tools, which is illustrated by the example of a two-level system.

Methodologically important is the "consistency (harmony) postulate" of goals the activities of management bodies of various levels aim to achieve. The fulfillment of this postulate is equivalent to the correct selection of goals and statement of problems for all management bodies that are part of the system. It also guarantees a reasonable combination of centralized and decentralized management of a large system. In this case, progress towards an overall goal can be achieved through appropriate coordination of the activities of subsystems, which are largely autonomous in terms of the way they operate. As long as the goals are compatible, the overall goal of the system and the goals of its subsystems are not contradictory, and the decisions taken at the lower levels correspond not to the overall goal, but to their own goals, which does not, nonetheless, prevent progress towards attaining the overall goal of the entire system.

The authors then present their own mathematical theory of coordination. The focus is on three possible principles of coordination: interaction prediction, interaction estimation; and interaction decoupling. The problem of modifications of objective functions for lower level subsystems in a two-level system which would allow coordination of the previously uncoordinated system is considered. Some iterative methods are provided to address the coordination problem. The applicability of the principles of coordination under different assumptions about the nature of the problems (problems of linear or convex programming, solved with the aid of the Dantzig–Wolfe decomposition, etc.) is analyzed. Possible ways to improve the performance of the system as a whole are considered.

The book is not free from certain shortcomings. One of the most important of them is that the problems considered by the authors are formulated at such a high level of generality that so far it is possible to obtain a constructive solution for them only for the simplest linear systems. Nevertheless, the book provides a useful ideology on the theoretical principles of building large systems with a hierarchical structure and managing such systems.

Among other scant fundamental publications that are available on the subject of the theory of hierarchical multilevel systems the monograph by Marvin Lee Manheim "Hierarchical structure: Model of Design and Planning Processes" [7] cannot go unnoticed. The introductory article to the Russian edition of the book contributed by Yu.V. Kovachich, B.M. Avdeev, and V.M. Levitsky [8] requires special consideration, which we will do after the presentation of the original book by M.L. Manheim.

In this book, the author uses the example of the highway location process to develop a problem-solving procedure in the form of a sequence of actions made up of one or more operators. Each operator has two main components: SEARCH: activities that generate a number of mutually exclusive operations, and SELECT: activities that result in developing a preference for one of the generated operations. Operators differ with respect to the cost of their implementation, the information about the solutions they implement, and their "level", i.e. in the level of granularity of the solutions. The proportion between levels allows to order the entire set of operators available for the decision-maker.

The concept of experiment is introduced, which means applying some operator to an operation performed previously so as to arrive at another operation. This new operation is the lower level one relative to the operation it is derived from, and it is included in the latter.

Each experiment requires a well-defined amount of resources. Operations identified as a result of some experiment and their costs are not known precisely. The goal of the decision-maker is to determine, at any stage of solving the problem, which experiment is most desirable at the next stage, taking into account the possible results of the experiment and the cost of performing it.

The model, which allows to identify the best experiment that is to be realized at the next stage, is formulated. The model is based on the Bayesian decision theory. It is assumed that the decision maker can attribute a subjective distribution of probabilities to each operation that was previously obtained. This probability distribution function is an a priori distribution. It is also assumed that each operator is characterized by its own distribution of conditional probabilities. For each given experiment, the distribution of probabilities of possible cost values of the generated operations is obtained from the distribution of similar probabilities for the previously performed operations to which the operator should be applied, and from the distribution of conditional probabilities for the given operator. The observed result of the experiment is the cost of the operation performed. Based on these results, previous distributions for one or more operations (selected according to specific rules) are adjusted according to the Bayes's rule.

In deciding which of the possible experiments to implement at the next stage, the task is to trade off the cost of carrying out the experiment with the benefits thereof, which is reflected in the search for solutions that would be less costly than the best solution found before. Thus, within the framework of the described probabilistic model, the expected cost criterion is used to find the best experiment.

Let us return to the above introductory article [8] to the Russian edition of the book by M.L. Manheim. It represents the methodology of the system design of complex engineering systems, which in turn represents the simultaneous development of both the control system, consisting of a number of subsystems, and the controlled object.

Let us consider a principle of the system design assuming that the general configuration of a system is established depending on constraints p_i that should be respected when designing the system. In addition to these constraints, the system is characterized by some evaluation function J (criterion) that serves as a measure of the advantage (the preference relation) that one variant of the system has over another.

The mathematical formulation of the problem in the form of an optimization problem raises no objections, but a number of significant circumstances hinder the direct solution of the problem. First of all, one should point out the lack of a priori information necessary to find the optimal variant of the system, because its characteristics as a set of parameters p_j that the designer can adjust to influence criterion evaluation J are unknown.

Owing to the aforesaid it makes sense to construct the procedure of designing a system in the form of a multistage process so that the volume of data on the system and the granularity of its representation at each stage would increase. However, among the entire set of available variants there are those unacceptable in terms of either the limitations imposed on the system or the objective function, hence they should be ruled out when considering the next stage. On the other hand, taking into account the details of the system representation, it may be necessary to "generate" its additional variants.

An approach of this kind to designing a system can be linked to some hierarchical model, where each level of the hierarchy is characterized by a certain depth of elaboration (granularity) of the system. In this case, the design process can be represented in the form of an appropriate sequence of operations on the hierarchical model ("decision tree"). Moreover, following the performance of the operation, it is necessary to refine the new distribution of probabilities for the variant evaluation criterion, which can be the cost of the implementation of the variant. The connection between a priori and a posteriori distributions can be established using the Bayes's rule.

Thus, the introductory article [8] supplements Manheim's book [7] in terms of methods of building a hierarchical structure for the design process and the use of Bayesian theory of decision-making for the purposeful selection of variants of the designed system.

Let us consider another approach to building a hierarchy of models of a complex system and preference criteria when choosing a variant of its design variants as presented in [9-11]. This approach is based on consideration of the tasks of external and internal design of a complex engineering system. At the stage of the external design the requirements to the main technical characteristics of the system are determined, which enables us to arrive at its defining, aggregated design parameters. Further detailing of the system appearance, designing subsystems and links between them, deciding on the parameters of specific elements of the system make up the process of the internal design.

The idea central to this approach is based on the assumption that the initial problem of the internal design in the form of a sufficiently detailed model of the system and the required full set of preference relations for all practical purposes is insoluble due to the huge dimensionality of the model and the multiplicity of preference relations, which oftentimes prove contradictory. Therefore, in [9-11] it is proposed to replace the initial problem that is deemed unfeasible with a hierarchy of sub-problems ("top-down") that grow in complexity from one step to the next one. Interrelation of sub-problems in this hierarchical structure is ensured by consistent aggregation of the system parameters in the bottom-up fashion along with the necessary transformation of preference relations (criteria).

The authors note that the practical implementation of the hierarchical approach can be represented, for example, by a major design studio (CB), in which each level of the -problems hierarchy is implemented by a dedicated designer. In this case, the problem of coordination of aggregation and preference relations on the set of sub-projects proves relevant. Consistency of preference and aggregation relations means that the designers of the k-th and $(k+1)$-th levels held approximately the same views on what makes a "good" engineering system that they are designing. This means that when moving from one level of aggregation to another, a higher one, no additional criteria for evaluating the engineering system are introduced. This condition explains away to some extent the mutual obligations of designers in hierarchical design systems of the design studio type. In the case of one designer in charge of all levels of the hierarchy, the above requirement disappears, but the authors do not cover this more general case.

It should be pointed out that this hierarchical approach actually merges the problems of external and internal designing of systems as it is expedient to formulate a sub-problems of the uppermost level of aggregation as a problem of the external designing.

The challenges related to information aggregation are addressed in [12] as applied to the task of planning in multilevel active systems, assuming that the control bodies of all levels are endowed with the property of activity, i.e. they have their own interests and pursue their own goals. In the case of decentralisation of the system, there is a need to aggregate information as the level of a hierarchy increases, so the following problem proves relevant: what are the decentralization options for the planning workflow that have aggregation as not contributing to compromising the properties of such a workflow, including, first of all, the efficiency of control. It is noted that there is no general solution to the problem of aggregation of active systems, so the article deals with a number of specific cases bearing on the transition from a two-level to a three-level active system.

A method of multi-criteria evaluation and optimization of hierarchical systems is proposed in [13]. The problem is formalized as an operation of choosing the preferred option on the set of options when using the vector-valued choice function F. The logic driving the vector-based approach requires decomposition of function F into a set (vector) of choice functions f. The study provides a justification for the claim that any multi-criteria problem can be represented by a hierarchical system. In doing so, at its lower level the evaluation of the object by individual properties with the help of the criteria vector is carried out, while at its upper level the evaluation of the object as a whole is achieved by means of the composition procedure. The central problem then is the composition of the criteria by hierarchy levels. To this end, a compromise-based framework is adopted. The method of solving complex multicriteria evaluation and optimization tasks based on consideration of nested scalar aggregates of vector criteria is proposed. At the same time, a hierarchy can be both natural (multi-level systems with top-down subordination) and that arising as a result of decomposition of the object properties down to the level of individual criteria (a hierarchy of criteria).

Published research [14, 15] offers hierarchical game-theoretical principles of how to study and control hierarchical systems. In [14], within the framework of the game-theoretical model of hierarchical control, which takes into account the requirements of sustainable development of the system, formalization of the required methods of hierarchical controls as specific instances of principles of optimality is performed. The Master and the Slave players are considered, with the Master employing the following methods in relation to the Slave: coercion; inducement; and persuasion. The basic principle of optimality is the Stackelberg equilibrium. In [15] a hierarchical structure is considered, in which there is a coordinating center (upper level) and coalition groups (lower level) that in addition to pursuing their own interests have to abide by the decisions of the center. The above study implements the principle of active equilibrium between coalitions, while the equilibrium in the hierarchy is Pareto-based under uncertainty of various kinds at both levels.

III. Conclusion

The research findings by various authors as expounded in this study detail in a sufficiently comprehensive way the general, in many respects overlapping, methodological views held on the subject of hierarchical systems, hierarchical modeling of large systems, and hierarchical control of such systems.

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Nikolai I. Voropai is Professor, Scientific Advisor of the Melentiev Energy Systems Institute of Siberian Branch of the Russian Academy of Sciences, Corresponding Member of the Russian Academy of Sciences. He received his Candidate of Engineering Sciences degree in 1974 and the Doctor of Engineering Sciences degree in 1990. (according to the Russian system of doctoral level scientific degrees). His research interests include modeling of power systems; operation, dynamics and control of large power grids; development of national, international, and intercontinental power grids; reliability and security of energy systems; power industry restructuring.