Reduction and Equivalencing of Equations of Electrical Network based on Matrix Annihilators

Magomed Gadzhiev*, Misrikhan Misrikhanov, Vladimir Ryabchenko and Yury Sharov

Institute of Electrical Power Engineering, National research university MPEI, Moscow, Russia

***Abstract*— This paper proposes an original method for the reduction of node voltage equations, aimed at equivalencing electrical network. The method is based on equivalent matrix transformations using matrix annihilators. The offered method, in comparison with the traditional one, makes it possible to improve the conditionality of the solved equations by an order of magnitude or more. This has a positive effect on the numerical stability of the resulting electrical network equivalents. The results of reduction of a small, large, and very large system of node voltage equations are presented.**

***Index Terms* — electrical network, nodal stress equations, reduction, matrix annihilator, equivalencing, conditionality, large and very large matrices.**

I. Introduction

It is well known that the calculation of electric power systems (EPS) conditions is multivariate. Solving the complete systems of equations covering all the nodes and connections of large power systems, even with simplified models, poses a serious problem, because of the lack of reliable information on all elements of the network. The problem is exacerbated by the need to accumulate and store large amounts of information [1] and the corresponding increase in the requirements for speed of computers [2].

The need to reduce the computation time arises mainly in multivariate and multimode calculations in operational control, and in EPS operation planning [3, 4]. Equivalencing makes it possible both to reduce the time of solving the node voltage equations (NVE), and to reconcile the amount of information and its error.

Equivalencing of EPS is the transformation (reduction) of a complex mathematical model into a simpler one while preserving the most important (required) properties within a given accuracy. This approach is widely used with the toolkit of Krylov subspaces [5]. There is one more approach to the calculation of EPS conditions, where equivalencing is reduced to the transformation of an equivalent circuit and its parameters to the one having a smaller number of nodes and branches and suitable for modeling of the initial EPS conditions.

The paper proposes an original NVE reduction method for equivalencing an electrical network. The method is based on the NVE matrix transformations with the help of algebraic objects, called annihilators of matrices.

This method, in comparison with the traditional algorithm based on the Schur complement, makes it possible to improve significantly the conditionality of the solved equations, especially for large (up to 10 000 equations) and very large (up to 100 000 equations) systems of NVE. It has a significant effect on the accuracy of the obtained electrical network equivalents and on the numerical correctness of numerical models.

II. The Traditional Approach To NVE Equivalencing

The procedure of equivalencing is given later as an example. Excluded nodes (set M) and nodes stored in equivalent units (set N) are given in [2].

The nodes were renumbered so that the first nodes were from the set N and the rest of the nodes were from the set M. In this case, the NVE were structurally divided into blocks (block matrices and subvectors)

$$\begin{bmatrix} \boldsymbol{Y}_{NN} & \boldsymbol{Y}_{MN} \\ \boldsymbol{Y}_{MN} & \boldsymbol{Y}_{MM} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_{N} \\ \boldsymbol{U}_{M} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{N} \\ \boldsymbol{I}_{M} \end{bmatrix}. \tag{1}$$

Here, $\boldsymbol{U}_N, \boldsymbol{U}_M$ – the voltage subvectors; $\boldsymbol{I}_N$, $\boldsymbol{I}_M$ – current sub-vectors in the vector

$$\begin{bmatrix} \boldsymbol{I}_{N} \\ \boldsymbol{I}_{M} \end{bmatrix}. \tag{2}$$

Submatrix $\boldsymbol{Y}_{NN}$ in the conductivity matrix

$$\begin{bmatrix} \boldsymbol{Y}_{NN} & \boldsymbol{Y}_{MN} \\ \boldsymbol{Y}_{MN} & \boldsymbol{Y}_{MM} \end{bmatrix} \tag{3}$$

is square and covers only the constraints on the set of nodes N, and the square submatrix $\mathbf{Y}_{\mathrm{MM}}$ corresponds to the

* Corresponding author.
E-mail: gadzhiev_mg@interrao.ru

http://dx.doi.org/10.25729/esr.2018.01.0011
Received: January 30, 2018. Revised: April 20, 2018. Accepted: April 21, 2018. Available online: April 25, 2018.

connections on the set *M*. Note that $\boldsymbol{Y}_{NM}$ and $\boldsymbol{Y}_{MN}$ are rectangular matrices.

Expanding equation (1) up to the selected blocks

$$\begin{aligned} \boldsymbol{Y}_{NN}\boldsymbol{U}_N + \boldsymbol{Y}_{NM}\boldsymbol{U}_M &= \boldsymbol{I}_N, \\ \boldsymbol{Y}_{MN}\boldsymbol{U}_N + \boldsymbol{Y}_{MM}\boldsymbol{U}_M &= \boldsymbol{I}_M, \end{aligned} \tag{4}$$

and expressing the voltage vector of the excluded nodes $\boldsymbol{U}_M$ from the second equation (4), we obtain the equation

$$\begin{aligned} &\boldsymbol{Y}_{NN} - \boldsymbol{Y}_{NM}\boldsymbol{Y}_{MM}^{-1}\boldsymbol{Y}_{MN} \;\; \boldsymbol{U}_N = \boldsymbol{I}_N - \\ &-\boldsymbol{Y}_{NM}\boldsymbol{Y}_{MM}^{-1}\boldsymbol{I}_M \end{aligned} \tag{5}$$

which with the introduction of new notations

$$\begin{aligned} \boldsymbol{Y}_{NN}^{э} &= \boldsymbol{Y}_{NN} - \boldsymbol{Y}_{NM}\boldsymbol{Y}_{MM}^{-1}\boldsymbol{Y}_{MN}, \\ \boldsymbol{I}_{N}^{э} &= \boldsymbol{I}_N - \boldsymbol{Y}_{NM}\boldsymbol{Y}_{MM}^{-1}\boldsymbol{I}_M, \end{aligned}$$

is reduced to the equivalent form

$$\boldsymbol{Y}_{NN}^{э}\boldsymbol{U}_N = \boldsymbol{I}_N^{э}. \tag{6}$$

Thus, the equivalencing procedure considered above, for the given excluded nodes (set *M*) preserves only the nodes of the set *N* in the resulting equivalent solution (equivalent).

From a formal point of view, the described equivalencing procedure is based on the well-known *Schur complement algorithm* [6, 7]. One of the drawbacks of this approach is the complexity and, often, the impossibility of preconditioning ("regulation" of the conditionality) of the solved equations.

The next section of the paper describes an original method. This method is an alternative to the *Schur complement algorithm*, and the equivalencing method is constructed on its basis.

This method, as was said earlier, makes it possible to effectively affect the NVE condition, and, as a consequence, reduce the computational errors and increase the correctness of the equivalent solutions obtained.

III. Mathematical Justification Of The Alternative Method

The following notations and definitions will be used: $\mathbf{0}_{n\times m}$ $- n\times m$ zero matrix; $\boldsymbol{E}_n$ $-$ $n\times n$ unit matrix; $(\cdot)^{\mathrm{T}}$ –transposed matrix; $(\cdot)^{+}$ –pseudo-inverse matrix according to Moore-Penrose; $(\cdot)^{\perp}$ – maximal rank matrix annihilator; $\mathrm{rank}(\cdot)$ – rank of the matrix; $\mathrm{size}(\cdot)$ – dimensions of the matrix (vector dimension); $\mathrm{null}(\cdot)$ – basis of the null space of the matrix; $\mathrm{cond}(\cdot)$ – condition number of the matrix; $||\cdot||$ –given vector norm [6, 7].

In this paper, the so-called left annihilator of matrices is used, which is called the annihilator. Recall [8, 9], that the maximal rank matrix annihilator of $m\times n$ matrix $\boldsymbol{M}$ of rank r is called matrix $\boldsymbol{M}^{\perp}$, and $\boldsymbol{M}^{\perp}\boldsymbol{M} = \mathbf{0}_{(n-r)\times m}$, with rank $\boldsymbol{M}^{\perp} = n - r$.

For simplicity, we shall assume that the annihilators of zero satisfy the orthogonality condition $\boldsymbol{M}^{\perp}\boldsymbol{M}^{\perp\mathrm{T}} = \boldsymbol{E}_{n-r}$.

Well-developed methods for computing the null space $\mathrm{null}(\boldsymbol{M})$ of a matrix $\mathbf{M}$ can be used for calculation of annihilator matrices [6, 10]. In this case $\boldsymbol{M}^{\perp} = \mathrm{null}\left(\boldsymbol{M}^{\mathrm{T}}\right)^{\mathrm{T}}$

We will consider the NVE in the following block decomposition:

$$\left[\boldsymbol{A}_1 \;\vdots\; \boldsymbol{A}_2\right]\begin{bmatrix}\mathbf{x}_1 \\ \mathbf{x}_2\end{bmatrix} = \mathbf{b}, \tag{7}$$

where $\boldsymbol{A}_1$, $\boldsymbol{A}_2$ – are rectangular submatrices of

$$\mathrm{size}\;\boldsymbol{A}_1 = n\times n_1, \quad \mathrm{size}\;\boldsymbol{A}_2 = n\times n_2, \tag{8}$$

in this case $n_1 + n_2 = n$. The decomposition (7), (8) is clearly shown in Fig. 1.

The statement: the solution of linear equation (7) for invertible block matrix $\left[\boldsymbol{A}_1 \;\vdots\; \boldsymbol{A}_2\right]$ is determined by the equivalent formulas [9]

$$\begin{cases} \mathbf{x}_1 = \boldsymbol{A}_1^{+} \;\; \mathbf{b} - \boldsymbol{A}_2\mathbf{x}_2 \;\; , \\ \mathbf{x}_2 = \;\; \boldsymbol{A}_1^{\perp}\boldsymbol{A}_2 \;\;^{-1} \boldsymbol{A}_1^{\perp}\mathbf{b}. \end{cases} \tag{9}$$

$$\begin{cases} \mathbf{x}_1 = \;\; \boldsymbol{A}_2^{\perp}\boldsymbol{A}_1 \;\;^{-1} \boldsymbol{A}_2^{\perp}\mathbf{b}, \\ \mathbf{x}_2 = \boldsymbol{A}_2^{+} \;\; \mathbf{b} - \boldsymbol{A}_1\mathbf{x}_1 \;\; , \end{cases} \tag{10}$$

here $\boldsymbol{A}_1^{\perp}$, $\boldsymbol{A}_2^{\perp}$ are the left annihilators of zero of the maximal rank submatrices $\boldsymbol{A}_1$, $\boldsymbol{A}_2$, respectively, $\boldsymbol{A}_1^{+}$, $\boldsymbol{A}_2^{+}$ are the pseudoinverse matrices of the submatrices $\boldsymbol{A}_1$, $\boldsymbol{A}_2$.

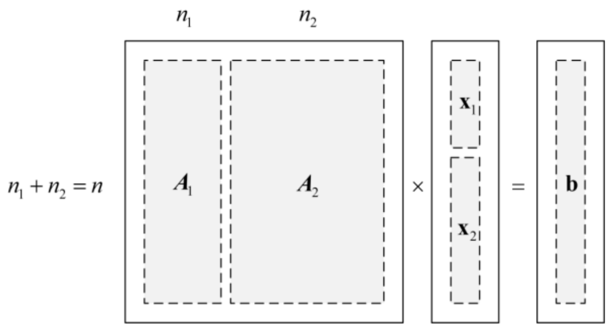


Figure 1. Block partition of matrices and vectors in the matrix equation.

IV. Reduction Of NVE Based On Matrix Annihilators

Considering NVE (1) in the division into blocks, as it is done in equation (7), we introduce the notations

$$\boldsymbol{Y}_N = \begin{bmatrix} \boldsymbol{Y}_{NN} \\ \boldsymbol{Y}_{MN} \end{bmatrix}, \quad \boldsymbol{Y}_M = \begin{bmatrix} \boldsymbol{Y}_{MN} \\ \boldsymbol{Y}_{MM} \end{bmatrix} \tag{11}$$

and write (1), taking into account (11). The obtained equation is

$$\begin{bmatrix} \boldsymbol{Y}_N & \boldsymbol{Y}_M \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_N \\ \boldsymbol{U}_M \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_N \\ \boldsymbol{I}_M \end{bmatrix}. \tag{12}$$

Annihilator $\boldsymbol{Y}_M^{\perp}$ is introduced and it satisfies the following conditions:

$$\boldsymbol{Y}_M^{\perp}\boldsymbol{Y}_M = \mathbf{0}_{M\times M}, \quad \boldsymbol{Y}_M^{\perp}\boldsymbol{Y}_M^{\perp\mathrm{T}} = \boldsymbol{E}_{M\times M}. \tag{13}$$

Then, according to the first equation (10), from the theorem proved earlier we can write

$$\boldsymbol{Y}_M^{\perp}\boldsymbol{Y}_N\boldsymbol{U}_N = \boldsymbol{Y}_M^{\perp}\begin{bmatrix} \boldsymbol{I}_N \\ \boldsymbol{I}_M \end{bmatrix}. \tag{14}$$

Let us introduce new notations

$$\widehat{\boldsymbol{Y}}^{\text{э}}_{NN} = \boldsymbol{Y}_M^{\perp}\boldsymbol{Y}_N, \quad \widehat{\boldsymbol{I}}^{\text{э}}_N = \boldsymbol{Y}_M^{\perp}\begin{bmatrix} \boldsymbol{I}_N \\ \boldsymbol{I}_M \end{bmatrix}, \tag{15}$$

then, equation (14) can be rewritten in a generalized form

$$\widehat{\boldsymbol{Y}}^{\text{э}}_{NN}\boldsymbol{U}_N = \widehat{\boldsymbol{I}}^{\text{э}}_N. \tag{16}$$

Equations (6) and (16) are different, but they have the same solution $\boldsymbol{U}_N$. The principal difference of equation (16) is that it allows solving the problem of preconditioning in order to minimize computational errors simultaneously with equivalencing.

It is well known [6, 11, 12] that in order to reduce the influence of errors in the initial data, to increase the accuracy of the solution, and to accelerate the convergence of the iterative methods, various algorithms are used that usually consist of elementary transformations of rows (columns) of matrices in equation (7): scaling, regularization, balancing, change of conditioning (preconditioning, use of spectrally equivalent operators), etc.

With respect to matrix equation (16), the problem of reducing errors will consist in minimizing the ratio [11]

$$\tau = \frac{\left\|\Delta\boldsymbol{U}_N\right\|\cdot\left\|\widehat{\boldsymbol{I}}^{\text{э}}_N\right\|}{\left\|\boldsymbol{U}_N\right\|\cdot\left\|\Delta\widehat{\boldsymbol{I}}^{\text{э}}_N\right\|}. \tag{17}$$

However, the direct determination of the value τ in terms of the coefficients of the matrices of the original equation is difficult due to the nonlinearity of the valuation operation. Therefore, it is preferable to use a qualitative characteristic called the matrix condition number [6, 10, 11]. In the considered case, this number is

$$\operatorname{cond}\left(\widehat{\boldsymbol{Y}}^{\text{э}}_{NN}\right) = \left\|\widehat{\boldsymbol{Y}}^{\text{э}}_{NN}\right\|\cdot\left\|\left(\widehat{\boldsymbol{Y}}^{\text{э}}_{NN}\right)^{-1}\right\| \tag{18}$$

and it satisfies the inequality

$$\frac{\left\|\Delta\boldsymbol{U}_N\right\|}{\left\|\boldsymbol{U}_N\right\|} \le \operatorname{cond}\left(\widehat{\boldsymbol{Y}}^{\text{э}}_{NN}\right) \le \frac{\left\|\Delta\widehat{\boldsymbol{I}}^{\text{э}}_N\right\|}{\left\|\widehat{\boldsymbol{I}}^{\text{э}}_N\right\|}. \tag{19}$$

Given (15), the ratio (19) is transformed to the form

$$\frac{\left\|\Delta\boldsymbol{U}_N\right\|}{\left\|\boldsymbol{U}_N\right\|} \le \operatorname{cond}\left(\boldsymbol{Y}_M^{\perp}\boldsymbol{Y}_N\right) \le \frac{\left\|\Delta\left(\boldsymbol{Y}_M^{\perp}\begin{pmatrix} \boldsymbol{I}_N \\ \boldsymbol{I}_M \end{pmatrix}\right)\right\|}{\left\|\boldsymbol{Y}_M^{\perp}\begin{pmatrix} \boldsymbol{I}_N \\ \boldsymbol{I}_M \end{pmatrix}\right\|}. \tag{20}$$

The larger the condition number (18), the greater the impact of the original data errors on the solution to NVE.

Reduction of the condition number (18) can be achieved by further transformation of the NVE equivalent system (16) by introducing a new matrix $\boldsymbol{D}$, which should be [12] as close as possible to $\left(\boldsymbol{Y}_M^{\perp}\boldsymbol{Y}_N\right)^{-1}$, easily computable and easily invertible. In this case, the NVE will be replaced by the equation

$$\boldsymbol{D}\widehat{\boldsymbol{Y}}^{\text{э}}_{NN}\boldsymbol{U}_N = \boldsymbol{D}\,\widehat{\boldsymbol{I}}^{\text{э}}_N,$$

where

$$\operatorname{cond}\left(\boldsymbol{D}\widehat{\boldsymbol{Y}}^{\text{э}}_{NN}\right) < \operatorname{cond}\left(\widehat{\boldsymbol{Y}}^{\text{э}}_{NN}\right).$$

V. Reduction Of A Small NVE System

Let us consider the computational example [9].

We analyze the electric network shown in Fig. 2 [2]. For convenience of calculation, we take all the line resistances to be the same $r_{ij} = 10\,\text{Ohm}$ ($Y_{ij} = 0{,}1$ S), except for the two lines $r_{13} = 20\,\text{Ohm}$ ($Y_{13} = 0{,}05$ S), $r_{24} = 5\,\text{Ohm}$ ($Y_{24} = 0{,}2$ S).

The basic mode corresponds to the NVE system

$$\begin{bmatrix} -0{,}25 & 0{,}1 & 0{,}05 & 0 \\ 0{,}1 & -0{,}4 & 0{,}1 & 0{,}2 \\ 0{,}05 & 0{,}1 & -0{,}25 & 0 \\ 0 & 0{,}2 & 0 & -0{,}2 \end{bmatrix} \begin{bmatrix} U_1^0 \\ U_2^0 \\ U_3^0 \\ U_4^0 \end{bmatrix} = \begin{bmatrix} -8{,}5 \\ 1 \\ -7 \\ -2 \end{bmatrix}, \qquad (21)$$

The results of the calculation are shown in Fig. 2.

Let us suppose that the set *M* of excluded nodes is 3 and 4. Then, matrix (3) has the following block decompositions:

$$\left[\boldsymbol{Y}_N \,\vdots\, \boldsymbol{Y}_M\right] = \left[\begin{array}{cc:cc} -0{,}25 & 0{,}1 & 0{,}05 & 0 \\ 0{,}1 & -0{,}4 & 0{,}1 & 0{,}2 \\ 0{,}05 & 0{,}1 & -0{,}25 & 0 \\ 0 & 0{,}2 & 0 & -0{,}2 \end{array}\right], \qquad (22)$$

considering that

$$\boldsymbol{Y}_M^{\perp} = \left(\text{null} \begin{bmatrix} 0{,}05 & 0 \\ 0{,}1 & 0{,}2 \\ -0{,}25 & 0 \\ 0 & -0{,}2 \end{bmatrix}^{\mathrm{T}} \right)^{\mathrm{T}} =$$

$$= \begin{bmatrix} 0{,}9646 & 0{,}0915 & 0{,}2295 & 0{,}0915 \\ -0{,}1837 & 0{,}6752 & 0{,}2333 & 0{,}6752 \end{bmatrix}$$

we have

$$\boldsymbol{Y}_M^{\perp}\boldsymbol{Y}_M = \begin{bmatrix} 0{,}9646 & 0{,}0915 & 0{,}2295 & 0{,}0915 \\ -0{,}1837 & 0{,}6752 & 0{,}2333 & 0{,}6752 \end{bmatrix}. \cdot \begin{bmatrix} 0{,}05 & 0 \\ 0{,}1 & 0{,}2 \\ -0{,}25 & 0 \\ 0 & -0{,}2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Performing further calculations, we obtain

$$\widehat{\boldsymbol{Y}}^{\text{э}}_{NN} = \boldsymbol{Y}_M^{\perp}\boldsymbol{Y}_N = \begin{bmatrix} -0{,}2205 & 0{,}1011 \\ 0{,}1251 & -0{,}1301 \end{bmatrix},$$

$$\widehat{\boldsymbol{I}}^{\text{э}}_N = \boldsymbol{Y}_M^{\perp} \begin{bmatrix} \boldsymbol{I}_N \\ \boldsymbol{I}_M \end{bmatrix} = \begin{bmatrix} -9{,}8979 \\ -0{,}7473 \end{bmatrix},$$

$$\boldsymbol{U}_N = \left(\widehat{\boldsymbol{Y}}^{\text{э}}_{NN}\right)^{-1} \widehat{\boldsymbol{I}}^{\text{э}}_N = \begin{bmatrix} 85{,}0 \\ 87{,}5 \end{bmatrix},$$

which exactly corresponds to the values indicated in Fig. 2.

It is worth noting, that the condition number of matrix (3) and the conductivity number of original matrix (22) is $\text{cond}\left[\boldsymbol{Y}_N \,\vdots\, \boldsymbol{Y}_M\right] = 15{,}9373$ for the original matrix, and

$$\text{cond}\left(\widehat{\boldsymbol{Y}}^{\text{э}}_{NN}\right) = \text{cond}\begin{bmatrix} -0{,}2205 & 0{,}1011 \\ 0{,}1251 & -0{,}1301 \end{bmatrix} = 5{,}5204,$$

for the equivalent, which is less by almost 3 times. This number can be further reduced if $\boldsymbol{Y}_M^{\perp}$ is

$$\boldsymbol{Y}_M^{\perp} = \begin{bmatrix} 6{,}6150 & 4{,}7836 & 3{,}2365 & 4{,}7836 \\ 3{,}5836 & 8{,}6518 & 4{,}1774 & 8{,}6518 \end{bmatrix} \qquad (23)$$

The use of annihilator (23) in the calculations provides

$$\widehat{\boldsymbol{Y}}^{\text{э}}_{NN} = \begin{bmatrix} -0{,}2205 & 0{,}1011 \\ 0{,}1251 & -0{,}1301 \end{bmatrix} \qquad (24)$$

$\text{cond}\left(\widehat{\boldsymbol{Y}}^{\text{э}}_{NN}\right) = 1{,}2444$, which is by more than an order of magnitude less than $\text{cond}\left[\boldsymbol{Y}_N \,\vdots\, \boldsymbol{Y}_M\right]$. In this case, the matrix

$$\boldsymbol{Y}^{\text{э}}_{NN} = \begin{bmatrix} -0{,}24 & 0{,}12 \\ 0{,}12 & -0{,}56 \end{bmatrix}.$$

calculated by the traditional method, has a 3,5 times greater condition number than matrix (24).

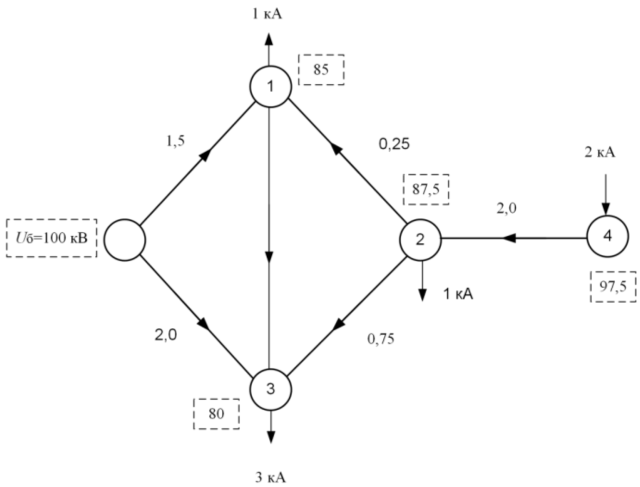


Figure 2. The scheme of the electrical network in the basic mode.

VI. Reduction Of A Large NVE System

Let us consider the reduction in a large NVE system, the basic mode of which corresponds to a matrix

$$\text{size}\begin{bmatrix} \boldsymbol{Y}_{NN} & \boldsymbol{Y}_{MN} \\ \boldsymbol{Y}_{MN} & \boldsymbol{Y}_{MM} \end{bmatrix} = 1000 \times 1000$$

and vector of

$$\text{size}\begin{bmatrix} \boldsymbol{I}_N \\ \boldsymbol{I}_M \end{bmatrix} = 1000 .$$

Matrix (3) and vector (2) for the basic mode are dense and their elements vary within the following limits: $-4,2; ...; 5,7$. The total number of non-zero elements of the matrix from (3) is $\sim 9,91 \cdot 10^5$.

Let the set *M* of excluded nodes be equal to 995, and, accordingly, the number of nodes $N = 1000 - 995 = 5$.

Calculation by formulas (11) – (16) in *Matlab*, using orthogonal annihilators leads to the following results:

$$\widehat{\boldsymbol{Y}}^{э}_{NN} = \begin{bmatrix} 1,8571 & -0,7800 & 1,4434 & 0,4049 & -0,4329 \\ -0,7242 & 0,6592 & 1,5200 & 0,5827 & 1,6713 \\ 0,2350 & -0,3492 & -0,6982 & 0,9702 & -1,3692 \\ -0,6484 & -0,6965 & 0,0061 & -1,1241 & 0,5262 \\ -0,8512 & 0,2244 & 0,1312 & 1,1808 & 1,2175 \end{bmatrix}, \quad (25)$$

$$\widehat{\boldsymbol{I}}^{э}_{N} = \left[0,7807; -0,1045; -1,4519; -0,2478; \ -0,1782\right],$$

$$\boldsymbol{U}_N = \left(\widehat{\boldsymbol{Y}}^{э}_{NN}\right)^{-1} \widehat{\boldsymbol{I}}^{э}_{N} = \begin{bmatrix} 1,6907 \\ 0,4469 \\ -0,8643 \\ -0,3758 \\ 1,4110 \end{bmatrix}. \quad (26)$$

In this case, the condition number of matrix (25) is 7,8019, and the Euclidean error rate of solution (26) with respect to the exact value of the solution vector is 3,825310^{-13}.

Calculations using conventional methods allow us to obtain the following matrix and vector

$$\boldsymbol{Y}^{э}_{NN} = \begin{bmatrix} 41,7122 & 19,8072 & -2,6691 & 84,1475 & -29,5413 \\ -55,6173 & -48,4439 & 9,7584 & -51,1095 & 43,1407 \\ -84,5763 & -19,1018 & 14,5854 & -42,1711 & 75,9152 \\ 87,8123 & 63,1374 & -21,0482 & 19,9958 & -59,6218 \\ 49,3494 & -8,4865 & 36,5263 & 64,3591 & -27,9839 \end{bmatrix} \quad (27)$$

$$\boldsymbol{I}^{э}_{N} = \begin{bmatrix} 8,3722 \\ -44,0346 \\ -41,1712 \\ 103,2302 \\ -15,6018 \end{bmatrix},$$

with more than 3,5 times the Euclidean norm of the error with respect to the exact value of vector 1,465110^{-12}. The condition number of matrix (27) is 39,4146 and it is almost 5 times higher than the condition number of matrix (25).

VII. Reduction In A Very Large NVE System

Let us suppose that a very large NVE system is given, whose dimension is 10^4. Matrix (3) has no zero elements, thus the number of non-zero elements is 10^8, that is one hundred million ($100 \cdot 10^6$). In this case, the elements in the matrix and vector (2) vary in the range from $-10,7$ to $11,5$. Suppose that the set of excluded nodes *M* is equal to $9 \cdot 10^3$, which means that the number of nodes left is $N = 10^4 - 9 \cdot 10^3 = 100$.

As a result of the calculations, the 100×100 matrix was obtained and it had the condition number $\text{cond}\left(\widehat{\boldsymbol{Y}}^{э}_{NN}\right) = 47,3464$, while the condition number of matrix $\boldsymbol{Y}^{э}_{NN}$ with the same size, was $\text{cond}\left(\boldsymbol{Y}^{э}_{NN}\right) = 10390,4112$, that is more than 200 times higher.

We note that it is not possible to directly use the Schur complement algorithm for such a large matrix. In this case, the authors used the parallelization of the computation process.

VIII. Conclusion

An original equivalencing algebraic method is proposed to reduce the equations of electric network steady-state conditions on the basis of matrix annihilators. The method allows transformations of the equivalent circuit and its parameters to a form having a significantly smaller number of nodes and branches. Numerical procedures for computing matrix annihilators are well developed for large ($100 \le n \le 1000$) and very large ($10^3 \le n \le 10^5$) matrices, while the variation in the annihilators properties makes it possible to significantly (ten times and more) improve the conditionality of the resulting NVE equivalents and thereby reduce computational errors and improve the correctness of the solution.

Acknowledgment

This work was supported financially by the Russian Science Foundation (Project №18-19-00225).

References

[1] Yu N., Shah S., Johnson R., Sherick R., Hong M., Loparo K. Big, “Data Analytics in Power

Distribution Systems", *Innovative Smart Grid Technologies Conference (ISGT), IEEE Power System*, 2015. [Online]. Available: https://pdfs.semanticscholar.org.

[2] B.I. Ayuev, P.I. Bartolomey, *Calculations of steady-state regimes in the tasks of operational and automatic EPS control*, Ekaterinburg, publishing house UTI-UPI, 1999, 40 p.

[3] M.Sh. Misrikhanov, V.N. Ryabchenko, "A method for solving algebraic systems with locally variable matrices", *Automation and Remote Control*, no.5, pp. 133 – 141, 2006.

[4] P.I. Bartolomey, A.V. Pasedin, "Automated dispatch control systems and optimization of EPS regimes", Ekaterinburg, publishing house UGTI-UPI, 1999.

[5] N.E. Zubov, E.A. Mikrin, V.N. Ryabchenko, *Matrix methods of automatic control systems for aircraft in the theory and practice*, Moscow: Publishing house of the Moscow State Technical University. Bauman, 2016, 666.

[6] J. Golub, C. Van Lown, *Matrix calculations*, Moscow: Mir, 1999, 548 p.

[7] Gantmakher F.R, *Theory of matrices*, Moscow: Nauka, 1967, 576 p.

[8] M.Sh. Misrikhanov, V.N. Ryabchenko. "Algebraic and matrix methods in the theory of linear MIMO-systems", *Vestnik IEEU*, no.5, pp, 196 – 240, 2005.

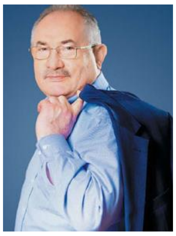
[9] M.Sh. Misrikhanov, V.N. Ryabchenko. "A method for the equivalence of an electrical network on the basis of matrix zero divisors", *Elektro*, no.1, pp.15 – 19, 2010.

[10] J. Demmel, *Computational linear algebra. Theory and applications*, Moscow: Mir, 2001, 430 p.

[11] Ikramov Kh.D, *Numerical solution of matrix equations*, Ed. D.K. Faddeeva, Moscow: Nauka, 1984, 192 p.

[12] M.Yu. Balandin, M.P. Shurina, *Methods for solving SLAE of large dimension*, Novosibirsk: Publishing house of the NSTU, 2000, 65 p.

Magomed Gadzhiev graduated from the Electric Power Department (EPD) of Moscow Power Engineering Institute (MPEI) in 2006. In 2012 he defended his PhD thesis "Improving the accuracy of accounting for power losses to the crown during optimization of EPS operating conditions" in MPEI. Currently he is the Head of the Research Laboratory of the Department of Electric Power Systems at MPEI.



Misrikhan Misrikhanov graduated from the Electric Power Department (EPD) of Moscow Power Engineering Institute (MPEI) in 1974. In 2010 he defended his doctoral dissertation «Fundamentals of the theory of invariant control and its application to the creation of technical systems». Currently he is a Professor at Ivanovo State University.

Vladimir Ryabchenko graduated from Irkutsk High Military Aviation Engineering School in 1983 with a specialty in «Aeronautical Instrument and Electrical Equipment». In 2002 he defended his doctoral dissertation in Air Force Academy named after Professor N.E. Zhukovsky. Leading researcher of the Research Laboratory «Automation of Electrical Distribution Networks» National Research University MPEI.

Yuriy Sharov graduated from the EEF MPEI in 1986. In 1994 he defended his PhD thesis "Development of methods and tools for assessing the effectiveness of the management of electric power systems under large disturbances" in MPEI. Currently he is the Head of the Department of Electric Power Systems at MPEI.