

# Methods and Algorithms for Selecting Independent Techno-Economic Indicators

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**Abstract** — One of the main requirements imposed on integrated indices is the independence of their components. In actual practice, a set of reliability, cost-effectiveness and safety indices of electric power facilities is known. It is based on real opportunities for data collection. The number of such technical and economic indicators varies from units to tens. If necessary, the informational content of the integrated estimates can be enhanced by involving the data on ratings and operation conditions. The joint use of these data, however, often encounters difficulties connected with the difference in both the scale of their measurement and the extent to which they are interrelated. The methods, algorithms, and subprograms are developed to select independent technical and economic indicators that increase the reliability of comparison and ranking of the facilities and to make recommendations for the improvement of the reliability and cost-effectiveness of their operation.

**Index Terms** — Reliability, efficiency, technical and economic indicators, ranking, boiler, distribution function

## I. INTRODUCTION

Operating experience with the equipment and devices (facilities) of electrical power systems (EPS) shows that the need for the reliability, cost-effectiveness, and safety (efficiency) of their operation has increased over time [1]. With the marked discrepancy between energy characteristics of the facilities and their technical condition, the main directions to improve the management of overall performance are to transition from intuitive

comparison and ranking of the EPS facilities to the automated management. A necessary condition for the automated management is the development of methods for calculation of operational values of the integrated indices (II).

The integrated index suggests independence of the technical and economic indicators (TEI) defining it. Violation of this condition distorts the magnitude of the integrated index depending on the number of the interconnected TEI and their relationships. Technical and economic indicators of EPS facilities have, as a rule, a quantitative scale of measurement. Their relationship is established by Pearson linear correlation coefficients  $\gamma_{op}$ , calculated based on the statistical data of operation and comparing  $\gamma_{op}$  with critical value  $\gamma_{\alpha}$  for the set type I error [2]. At the same time, it is supposed that the distribution of  $F(\text{TEI})$  corresponds to the normal law and the number of sample realizations  $n_s > 30$ . This assumption underlies the calculation of  $\gamma$ , provides the correspondence of distribution of  $F(\gamma)$  to the normal law and the possibility of assessing the accuracy (boundary values of a confidence interval) of the estimation of  $\gamma_{op}$ .

In actuality, however, the data on all techno-economic indicators characterizing the technical condition of a facility are not always available, the number of sample realizations (for example, the number of the same-type energy units) is not always  $n_s > 30$ , the distribution of  $F^*(\text{TEI})$  does not always correspond to the normal law, and relationships between techno-economic indicators are not always linear. This discrepancy raises doubts about the reliability of the analysis of TEI relationship, with all that it implies. *The methods to assess the relationships between real TEI samples are not developed.* The informational content of integrated indices can if necessary be enhanced by attracting, for example, some rated data characterizing overall performance of a certain facility. These data, however, have a rank-order scale of measurement, which creates certain difficulties for direct calculation of integrated indices following from the difference in the scales of measurement. However, given that data on TEI

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of the facilities measured with a numerical scale can be transformed so that it will be possible to measure them with a rank-order scale (and not vice versa), this difficulty of assessing the relationship between the considered indicators of the facility will be partially overcome [3].

The existence of a relationship between the indicators measured on a rank-order scale in practice is controlled by one of the rank tests. For the analysis, we have chosen the Spearman rank correlation coefficient, which is described in detail in [4].

1. For the automated comparison and ranking of the efficiency of the electric power system facilities, it is necessary to be able to calculate their integrated indices of operation in terms of single indicators that have different units, scales, and scale of measurement.
2. The accuracy of the integrated indices calculation depends considerably on the number of the interconnected single indicators  $m_p$ : the larger the number of  $m_p$ , the larger the calculation error.
3. The existence of relationship between techno-economic indicators is traditionally established based on the excess of the correlation coefficient calculated according to the statistical data on facility operation over a critical value with the set type I error  $\alpha$ .
4. The selection of functionally and statistically independent TEI allows us not only to increase the reliability of the integrated index evaluation but also to reduce the volume of necessary information and bulkiness of calculations.

## II. METHOD AND ALGORITHM FOR MODELING INDEPENDENT REALIZATIONS OF SPEARMAN RANK CORRELATION COEFFICIENT

Traditionally, for small  $n_s$  of TEI of ranked facilities (for example, generating units of power plants), the significance of the Spearman rank correlation coefficient  $\rho_{op}$  is determined by:

- Calculating Student's t-test by the expression

$$t_3 = \frac{\rho_{op} \sqrt{n_s - 2}}{\sqrt{1 - \rho_{op}^2}} \quad (1)$$

- Determining a critical value of Student's t-test  $t_\alpha$  for a set level of significance  $\alpha$  and the number of degrees of freedom ( $n_s - 2$ ) from the reference book;

- Comparing the values of  $t_{op}$  and  $t_\alpha$  :

$$\text{if } t_{op} > t_\alpha, \text{ then } H \Rightarrow H_2, \text{ else } H \Rightarrow H_1 \quad (2)$$

where  $H$  - the assumption;  $\Rightarrow$  - compliance;  $H_1$  and  $H_2$  - assumptions (hypotheses) about the absence or existence of a significant relationship between TEI.

Compilation of a list of independent TEI is an intermediate stage for comparing and ranking the facilities, therefore the manual calculation of a set of the values  $\rho_{op}$  (for example, when the number of TEI equals 10, it is necessary to rank 90  $\rho_{op}$  values) causes awkwardness, labor input and high risk of wrong decision.

We have developed an automated system for comparing and ranking the EPS facilities, which makes it possible both to perform faultless calculations of  $\rho_{op}$ , and to determine critical  $\rho_\alpha$  values directly by the statistical function of distribution (SFD) of  $F^*(\rho_M)$ . Modeling of possible realizations of  $\rho_M$  and formation of SFD  $F^*(\rho_M)$  were carried out as follows:

1. The standard RAND() program models two independent samples of random variables  $\{\xi_1\}_n$  and  $\{\xi_2\}_n$  with a volume of  $n_s$  that are uniformly distributed on the interval [0;1].

2. A sequence of serial numbers (ranks) of sample realizations  $\{\xi_1\}_n$  and  $\{\xi_2\}_n$  in terms of their variation series is formed. Let them be designated by  $\{r_1\}_n$  and  $\{r_2\}_n$ . Thus, the indicators measured with the numerical scale will be transformed into indicators with the rank-order scale of measurement.

3. The first realization of  $\rho_{1,M}$  is calculated according to Spearman's formula [4]

$$\rho_{1,i} = 1 - \frac{6 \cdot \sum_{j=1}^{n_s} (r_{1,j} - r_{2,j})^2}{n_s (n_s^2 - 1)} \quad (3)$$

4. Calculations for points 1-3 are repeated  $N$  times.

5. A variation series of a set of possible values  $\rho_{1,M}$  is built in ascending order of  $\rho_{1,n}$ . The probability  $F_1^*(\rho_{1,M}) = \frac{i}{N}$  is compared to each ordinal number of this series. At the same time  $\alpha = 1 - F^*(\rho_{1,M}) = R^*(\rho_{1,M})$ ,

Table 1. The sequence of calculation of  $\rho_{1,M}$ .

| Number of items | $\xi_{1,j}$ | $\xi_{2,j}$ | $r_{1,j}$ | $r_{2,j}$ | $\Delta r_j = r_{1,j} - r_{2,j}$ | $\Delta r_j^2$ | Note   |
|-----------------|-------------|-------------|-----------|-----------|----------------------------------|----------------|--|
| 1               | 0,287       | 0,249       | 1         | 1         | 0                                | 0              | $\rho_{1,i} = 1 - \frac{6 \cdot \sum_{j=1}^{n_s} \Delta r_j^2}{n_s (n_s^2 - 1)} = 0.7$ |
| 2               | 0,337       | 0,776       | 3         | 4         | -1                               | 1              |  |
| 3               | 0,806       | 0,265       | 4         | 2         | 2                                | 4              |  |
| 4               | 0,998       | 0,913       | 5         | 5         | 0                                | 0              |  |
| 5               | 0,303       | 0,633       | 2         | 3         | -1                               | 1              |  |
| Total           |             |             | 15        | 15        | 0                                | 6              |  |

The sequence of  $\rho_{1,m}$  calculations using formula (3) is given in Table 1 for illustrative purposes.

As one would expect, we will receive similar results with calculations based on expression (4) that represents a nonparametric analog of formula for calculation of Pearson linear correlation coefficient:

$$\rho_{1,op} = \frac{\sum_{j=1}^{n_s} (r_{1,j} - \bar{r}_1) \cdot (r_{2,j} - \bar{r}_2)}{\sqrt{\sum_{j=1}^{n_s} (r_{1,j} - \bar{r}_1)^2} \cdot \sqrt{\sum_{j=1}^{n_s} (r_{2,j} - \bar{r}_2)^2}} = 0,7 \quad (4)$$

where  $\bar{r}_1$  and  $\bar{r}_2$  are medians of samples  $\{r_1\}_{n_s}$  and  $\{r_2\}_{n_s}$ , respectively.

Calculations of  $\gamma_{op}$  by Pearson's formula

$$\gamma_{op} = \frac{\sum_{j=1}^{n_s} (\xi_{1,j} - \bar{\xi}_1) \cdot (\xi_{2,j} - \bar{\xi}_2)}{\sqrt{\sum_{j=1}^n (\xi_{1,j} - \bar{\xi}_1)^2} \cdot \sqrt{\sum_{j=1}^n (\xi_{2,j} - \bar{\xi}_2)^2}} = 0,252 \quad (5)$$

confirms possible essential distinction between linear ( $\gamma_{op}$ ) and rank ( $\rho_{op}$ ) correlation coefficients for the same samples  $\{\xi_1\}_n$  and  $\{\xi_2\}_n$ . It is worth reminding that random variables  $\xi$  have a uniform distribution on the interval [0,1], i.e. do not correspond to the normal law of distribution, which is the initial prerequisite for the application of Pearson's formula (5).

### III. CONSIDERATION OF IDENTICAL REALIZATIONS OF INDICATORS OF ATTRIBUTES

The above-discussed method for transformation of a numerical scale of measurement of random variables  $\xi$  to a rank-order scale does not allow for a degree of their divergence.

At a fixed number of intervals equal to  $r_{max}=5$  and the number of realizations of the modeled samples  $n_s \geq 5$ , the existence of repeated numbers of intervals (points) is inevitable. This statement is confirmed by the data of Table 2 that demonstrates the distribution of the degree of influence of six attributes on reliability and cost-effectiveness of power transformers [3]. The degree of influence is specified in points of a five-point system.

To consider this feature, we will somewhat transform points 2 and 3 of the algorithm for construction of  $F^*(\rho_M)$ .

A range of change in  $\xi$  [0;1] is represented by five equal intervals: (0,0,2), (0,21-0,4), (0,41-0,6), (0,61-0,8), (0,81-1,0). For realization of samples, number b of the interval (the number of points corresponding to each realization) corresponding to them is determined. Thus, the samples  $\{\xi_1\}_n$  and  $\{\xi_2\}_n$  are replaced with the samples of points  $\{b_1\}_n$  and  $\{b_2\}_n$ . Unlike the samples of random variables  $\xi$ , the samples  $\{b_1\}_n$  and  $\{b_2\}_n$  may contain two and more identical values.

According to [4], we arrange samples  $\{b_1\}_n$  and  $\{b_2\}_n$  in ascending order, assign serial numbers (ranks) to realizations of the samples, identify identical realizations in samples, and calculate an average size of ranks for identical realizations.

Table 3 demonstrates the results of calculations and assessment of  $\rho_{2,m}$  by (4). Samples of random variables  $\{\xi_1\}_5$  and  $\{\xi_2\}_5$  are the same as in Table 1.

The comparison of Tables 1 and 3 shows that the estimates  $\rho_{1,op}$ , and  $\rho_{2,m}$  differ greatly. The value of  $\rho_{2,m}$  calculated by formula (4) is less than  $\rho_{1,op}$ , calculated by formula (3).

The block diagram of the algorithm for the formation of ranks of realizations of discrete random variables is presented in Fig.1.

Table 2. An assessment of the rate of occurrence of points characterizing the reliability of power transformers

| Number of items | Transformer type  | The frequency of emergence of point |   |   |   |   |
|-----------------|-------------------|-------------------------------------|---|---|---|---|
|                 |                   | 1                                   | 2 | 3 | 4 | 5 |
| 1               | АТДЦТН-250000/220 | -                                   | - | - | 4 | 2 |
| 2               | ТДТН-63000/110    | -                                   | 1 | 4 | 1 | - |
| 3               | IEC-60076         | -                                   | - | 3 | 1 | 1 |
| 4               | ТДН-16000/110     | 1                                   | 5 | - | - | - |
| 5               | ТДН-25000/110     | -                                   | - | - | 3 | 3 |
| 6               | ТДТН-25000/110    | 1                                   | 4 | 1 | - | - |
| 7               | ТДТН-40000/110    | 4                                   | 1 | 1 | - | - |
| 8               | ТДТН-25000/110    | -                                   | - | - | - | 6 |
| 9               | ТМН-10000/110     | -                                   | - | 3 | 1 | 2 |
| 10              | АТДЦТН-250000/330 | -                                   | 1 | - | 2 | 3 |
| 11              | ТДН-80000/330     | -                                   | - | - | 6 | - |
| 12              | ТДТН-10000/110    | 1                                   | 1 | 4 | - | - |
| 13              | ТДН-25000/110     | -                                   | 1 | 1 | 4 | - |
| 14              | АТДЦТН-200000/220 | 1                                   | 2 | 2 | - | 1 |
| 15              | IEC-60076         | -                                   | - | 2 | 3 | 1 |
| 16              | АОДЦТН-167000/500 | -                                   | - | - | 2 | 4 |
| 17              | АОДЦТН-167000/500 | -                                   | - | - | 2 | 4 |
| 18              | ТДТН-40000/110    | 2                                   | 1 | 1 | 1 | 1 |
| 19              | АТДЦТН-250000/220 | 1                                   | - | 1 | 2 | 3 |

Table 3. Order of  $\rho_{4,M}$  calculation by (4).

| $j$   | $b_{1,j}$ | $b_{2,j}$ | $r_{1,h}$ | $r_{2,j}$ | $\Delta r_{1,j} = r_{1,j} - \bar{r}_1$ | $\Delta r_{2,j} = r_{2,j} - \bar{r}_2$ | $\Delta r_{1,j} - \Delta r_{2,j}$ | $\Delta r_{1,j}^2$ | $\Delta r_{2,j}^2$ | Note  |
|-------|-----------|-----------|-----------|-----------|--|--|-----------------------------------|--------------------|--------------------|---|
| 1     | 2         | 2         | 2         | 1,5       | -1                                     | -1,5                                   | 1,5                               | 1                  | 2,25               | $\bar{r}_1 = \bar{r}_2 = 3$<br>$\rho_2 = 0,152$ |
| 2     | 2         | 4         | 2         | 3,5       | -1                                     | 0,5                                    | -0,5                              | 1                  | 0,25               |   |
| 3     | 5         | 2         | 2         | 3,5       | -1                                     | 0,5                                    | -0,5                              | 1                  | 0,25               |   |
| 4     | 5         | 5         | 4,5       | 1,5       | 1,5                                    | -1,5                                   | -2,25                             | 2,25               | 2,25               |   |
| 5     | 2         | 4         | 4,5       | 5         | 1,5                                    | 2                                      | 3                                 | 2,25               | 4                  |   |
| Total |           |           | 15        | 15        | 0                                      | 0                                      | 1,25                              | 7,5                | 9                  |   |

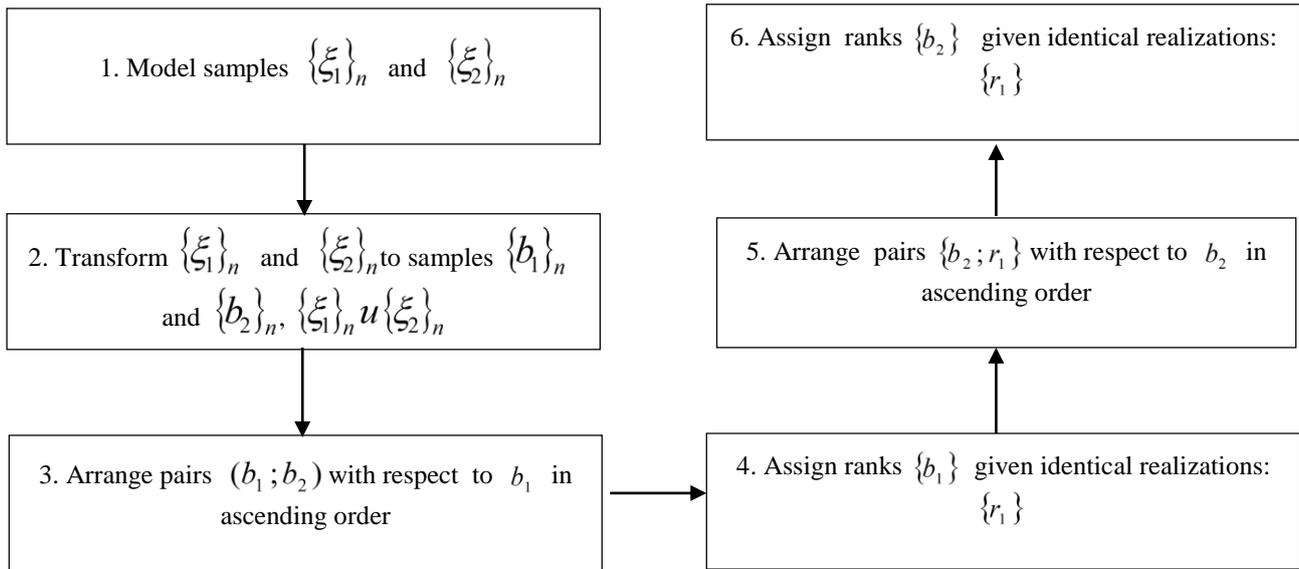


Figure 1. A block diagram of the algorithm for the formation of ranks of discrete random variables

IV. ANALYSIS OF FIDUCIAL DISTRIBUTIONS OF CORRELATION COEFFICIENTS OF THE CHOICE OF INDEPENDENT VALUES.

The above calculations and publications demonstrate that the relationship between Pearson  $\gamma$  and Spearman  $\rho$  correlation coefficients for the same samples is non-unique. At the same time, their critical values are assumed to be equal [6]. To analyze this feature, we propose comparing fiducial distributions of these correlation coefficients. It is worthwhile to remind that fiducial distribution means the distribution of possible realizations of integrated indices [7]. The integrated indices are taken to mean the indices whose realizations can be obtained from calculations. The integrated indices, for example, can be represented by arithmetic (geometric, harmonic) mean of random variables, indices of their spread, the coefficient of technical availability, and availability factor. The correlation coefficients also refer to them. This is a special approach to solving the inverse problem. Correlation coefficients are calculated for the samples of independent random variables providing their statistical independence.

A large number of calculations are required to obtain possible realizations of the correlation coefficients. For this reason, the automated system for the formation of the statistical functions of fiducial distributions (SFFDs) is necessary. The SFFD of each correlation coefficient can be

formed by repeatedly modeled samples, and all correlation coefficients can be calculated by one, but a repeatedly modeled pair of samples of random variables. The result of the SFFD calculation will be the same. However, the second way allows estimating the difference between the correlation coefficients. The developed algorithm and calculation program suggest the following sequence of calculations:

1. Two samples  $\{\xi_1\}_{n_s}$  and  $\{\xi_2\}_{n_s}$ , random variables  $\xi$  with uniform distribution in the range of [0,1] with the set  $n_s$  volume are modeled [8].
2. Pearson linear correlation coefficient  $\gamma_{1,M}$  is calculated by (5).
3. Samples  $\{\xi_1\}_{n_s}$  and  $\{\xi_2\}_{n_s}$  are transformed to the samples of points  $\{b_1\}_{n_s}$  and  $\{b_2\}_{n_s}$  for which Pearson linear correlation coefficients  $\gamma_{2,M}$  are calculated by (5).
4. Sample ranks  $\{r_1\}_{n_s}$  and  $\{r_2\}_{n_s}$  for samples  $\{\xi_1\}_{n_s}$  and  $\{\xi_2\}_{n_s}$  are determined. The ranks are used to calculate Spearman rank correlation coefficients  $\rho_{1,M}$  by (1).
5. Spearman rank correlation coefficient  $\rho_{2,M}$  is calculated for samples  $\{b_1\}_{n_s}$  and  $\{b_2\}_{n_s}$  by (4), given identical realizations.

Table 4. Relationship between realizations of linear and rank correlation coefficients for  $n_s=5$

| Numbers of items | Correlation coefficient at $n_s=5$ |                |              |              |
|------------------|------------------------------------|----------------|--------------|--------------|
|                  | $\gamma_{1,M}$                     | $\gamma_{2,M}$ | $\rho_{1,M}$ | $\rho_{2,M}$ |
| 1                | <u>0,404</u>                       | 0,319          | 0,2          | 0,306        |
| 2                | 0,612                              | 0,593          | <u>0,7</u>   | 0,574        |
| 3                | 0,213                              | <u>0,321</u>   | 0,1          | 0,162        |
| 4                | 0,346                              | 0,612          | -0,5         | <u>0,645</u> |
| 5                | -0,247                             | -0,339         | <u>-0,4</u>  | -0,34        |
| 6                | 0,521                              | 0,461          | 0,7          | <u>0,730</u> |
| 7                | <u>-0,340</u>                      | -0,240         | 0,1          | -0,278       |
| 8                | -0,925                             | <u>-0,955</u>  | -0,8         | -0,889       |
| 9                | 0,217                              | 0,036          | 0,2          | <u>0,263</u> |
| 10               | -0,881                             | -0,785         | -0,9         | -0,763       |

Table 5. The difference in solutions related to the relationship between samples for linear and rank correlation coefficients

| Numbers of items | Samples $\{\xi\}_5$ |       |       |       |       |       | Calculation results |                |              |               |
|------------------|---------------------|-------|-------|-------|-------|-------|---------------------|----------------|--------------|---------------|
|                  |                     |       |       |       |       |       | $\gamma_{1,M}$      | $\gamma_{2,M}$ | $\rho_{1,M}$ | $\rho_{2,M}$  |
| 1                | 1                   | 0,334 | 0,699 | 0,425 | 0,290 | 0,693 | 0,832               | <u>0,896</u>   | 0,8          | <u>0,973</u>  |
|                  | 2                   | 0,076 | 0,910 | 0,618 | 0,544 | 0,984 |                     |                |              |               |
| 2                | 1                   | 0,879 | 0,405 | 0,366 | 0,661 | 0,611 | <u>0,947</u>        | 0,832          | 0,8          | 0,872         |
|                  | 2                   | 0,965 | 0,136 | 0,265 | 0,563 | 0,709 |                     |                |              |               |
| 3                | 1                   | 0,059 | 0,688 | 0,434 | 0,281 | 0,872 | 0,750               | 0,808          | <u>0,9</u>   | 0,947         |
|                  | 2                   | 0,533 | 0,129 | 0,800 | 0,670 | 0,836 |                     |                |              |               |
| 4                | 1                   | 0,179 | 0,618 | 0,096 | 0,651 | 0,459 | -0,846              | <u>-0,885</u>  | <u>-0,9</u>  | <u>-0,884</u> |
|                  | 2                   | 0,486 | 0,396 | 0,674 | 0,204 | 0,256 |                     |                |              |               |
| 5                | 1                   | 0,867 | 0,404 | 0,433 | 0,394 | 0,790 | <u>-0,928</u>       | <u>-0,908</u>  | -0,7         | <u>-0,918</u> |
|                  | 2                   | 0,300 | 0,942 | 0,909 | 0,846 | 0,635 |                     |                |              |               |

Table 6. Critical values  $\Gamma_{1,M,\alpha}$ ,  $\Gamma_{2,M,\alpha}$ ,  $\rho_{1,M,\alpha}$  AND  $\rho_{2,M,\alpha}$  AT  $N_s=5$

| $\alpha$ | $\gamma_{1,M,\alpha}$ | $\gamma_{2,M,\alpha}$ | $\rho_{1,M,\alpha}$ | $\rho_{2,M,\alpha}$ | $\gamma_{1,M,\alpha}$ [6] |
|----------|-----------------------|-----------------------|---------------------|---------------------|---------------------------|
| 0,1      | 0,8120                | 0,8077                | 0,8                 | 0,8158              | 0,805                     |
| 0,05     | 0,8852                | 0,8847                | 0,9                 | 0,8922              | 0,878                     |
| 0,01     | 0,9620                | 0,9625                | 1,0                 | 0,9733              | 0,959                     |
| 0,005    | 0,9761                | 0,9715                | 1,0                 | 0,5747              | -                         |
| 0,001    | 0,9887                | 1,0                   | 1,0                 | 1,0                 | 0,991                     |

Table 7. Quantiles of distributions corresponding to  $F^*(...)=0,5$

|    | $\gamma_{1,M}$ | $\gamma_{2,M}$ | $\rho_{1,M}$ | $\rho_{2,M}$ |
|----|----------------|----------------|--------------|--------------|
| 5  | 0,4033         | 0,4082         | 0,4          | 0,3947       |
| 8  | 0,2754         | 0,2790         | 0,2613       | 0,2717       |
| 10 | 0,2325         | 0,2376         | 0,2364       | 0,2357       |

6. Points 1-5 were repeated  $N=10000$  times.

7. As a result:

7.1. For illustration, the relationship between realizations  $\gamma_{1,M}$ ,  $\gamma_{2,M}$ ,  $\rho_{1,M}$  and  $\rho_{2,M}$  are printed for the first ten pairs of samples  $\{b_1\}_{n_s}$  and  $\{b_2\}_{n_s}$  of their correlation

coefficients. The results of such calculations are given in Table 4 for  $n_s=5$ . The underlined figures mean the maximum absolute values of the correlation coefficients.

7.2. Corresponding samples  $\{\xi_1\}_{n_s}$  and  $\{\xi_2\}_{n_s}$ , and also correlation coefficients  $\gamma_{1,M}$ ,  $\gamma_{2,M}$ ,  $\rho_{1,M}$  and  $\rho_{2,M}$  were printed to illustrate the relationships between realizations of  $\gamma_{1,M}$ ,  $\gamma_{2,M}$ ,  $\rho_{1,M}$  and  $\rho_{2,M}$  provided that at least one of them exceeds the critical value given in literature data  $\gamma_\alpha=\rho_\alpha$ . Some characteristic results of modeling for  $n_s=5$  and  $\gamma_{0,05}=\rho_{0,05}=0,878$  are given in Table 5. The samples allow controlling reliability of the calculations.

7.3. The corresponding values  $\gamma_{1,M,\alpha}$ ,  $\gamma_{2,M,\alpha}$ ,  $\rho_{1,M,\alpha}$  and  $\rho_{2,M,\alpha}$  were determined by  $N_\alpha=(1-\alpha)N$  to compare the critical values of linear and rank correlation coefficients at  $\alpha$  equal to: 0,05; 0,01; 0,005; 0,001 0,1 and  $n_s=5$ . The results of the calculations, and reference data about  $\gamma_{1,M,\alpha}$  are given in Table 6.

7.4. Realizations of  $\gamma_{1,M}$ ,  $\gamma_{2,M}$ ,  $\rho_{1,M}$  and  $\rho_{2,M}$  were autonomously ranked and realizations corresponding to the probability of  $F^*(...)=0,05 \cdot i \cdot N$  at  $i=1,20$  were printed to compare the regularities of variations in SFFDs of possible realizations of linear and rank correlation coefficients. Quantiles of these SFFDs corresponding to the probability of  $F^*(...)=0,5$  and series  $n_s=5$  are given in Table 7.

Figure 2 demonstrates the histograms of  $f^*(\gamma_{1,M})$ ,  $f^*(\gamma_{2,M})$  and statistical functions of fiducial distribution of Pearson  $\gamma_1$  and Spearman  $\rho_2$  correlation coefficients for  $n_s=5$ .

The analysis of modeling allows us to draw the following conclusions.

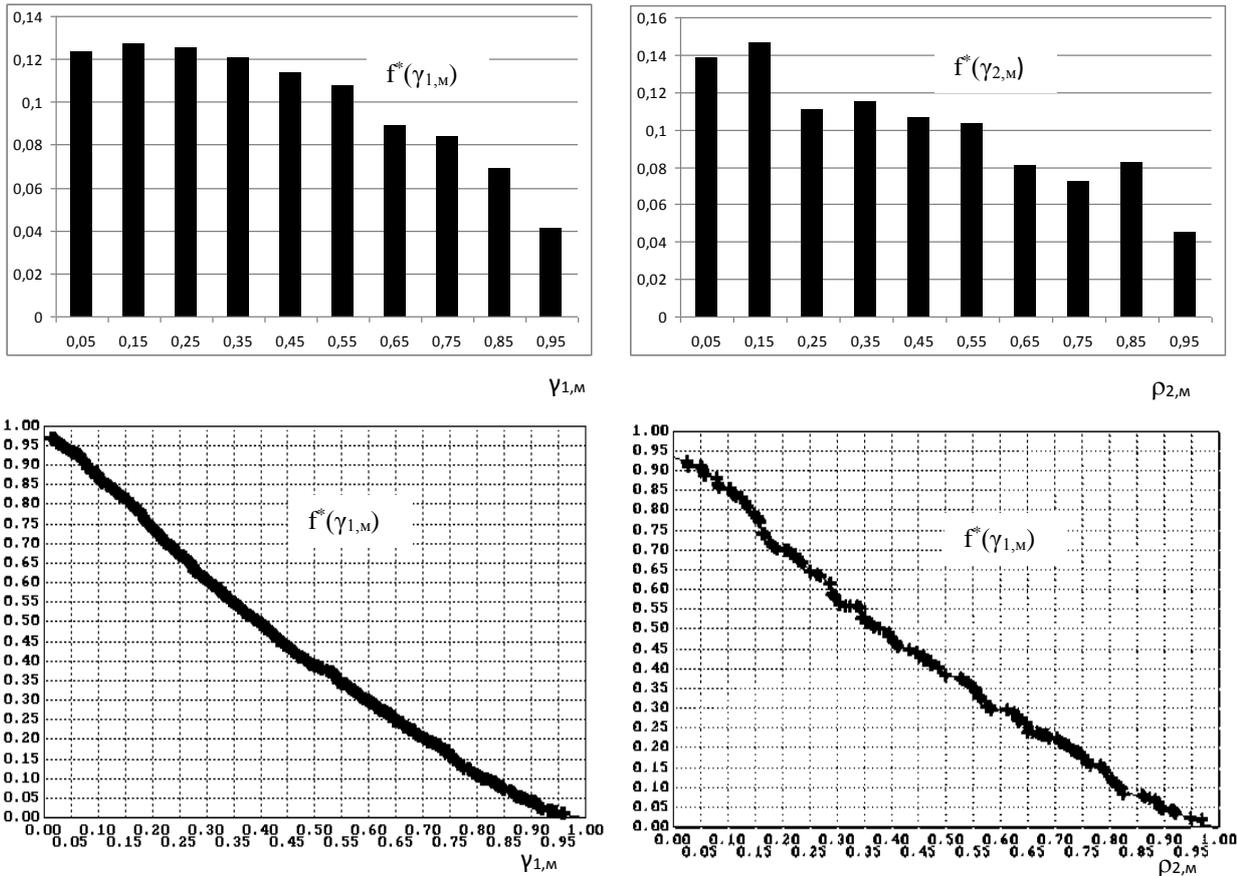


Figure 2. Histograms of distribution of correlation coefficients  $f^*(\gamma_{1,m})$ ,  $f^*(\rho_{2,m})$  and SFFDs  $R^*(\gamma_{1,m})$ ,  $R^*(\rho_{1,m})$  KK at  $n_s=5$ ;  $R^*(\gamma_{1,m})=1-F(\gamma_{1,m})$   $R^*(\rho_{1,m})=1-F(\rho_{1,m})$

Data from Table 4 indicate that the numerical estimates of correlation coefficients  $\gamma_1$ ,  $\gamma_2$ ,  $\rho_1$  and  $\rho_2$  for the same samples can differ greatly.

Table 5 highlights the largest values of the correlation coefficients for each of ten "tests". Pearson coefficients of linear correlation between realizations  $\gamma_{1,m}$ ,  $\gamma_{2,m}$ ,  $\rho_{1,m}$  and  $\rho_{2,m}$  exceed the critical value  $\gamma_{0,01}=0,765$ , which demonstrates the existence of a linear statistical relationship between them.

The data of Table 5 confirm the effectiveness of calculating no less than two correlation coefficients, in particular:

- When indicators are measured with a numerical scale, it is advisable to calculate Pearson linear correlation coefficients  $\gamma_1$  and  $\gamma_2$  and Spearman rank-order correlation coefficient  $\rho_2$  [9];
- When indicators are measured in points, it is better to calculate Pearson linear correlation coefficient  $\gamma_2$  and Spearman rank-order correlation coefficient  $\rho_2$ .

The recommendation is based on the difference in the extent to which these correlation coefficients take into account the properties and volume of samples (nature of distribution, the existence of identical realizations, small sample size, nonlinear nature of the relationship, etc.);

The data of Table 6 demonstrate almost insignificant divergence between the critical values of the linear

correlation coefficients  $\gamma_{1,m,\alpha}$ , and  $\gamma_{2,m,\alpha}$  obtained as a result of modeling, and theoretical data  $(\gamma_{1,\alpha})$ . It is worth noting that  $\gamma_{m,\alpha}$  are calculated by the samples whose random realizations have uniform distribution in the range of  $[0,1]$ . Although Pearson linear correlation coefficient suggests a correspondence between random variables of samples and the normal law, in the case of small  $n_s$ , the law of distribution has insignificant influence. The insignificant divergence between  $\gamma_{m,\alpha}$  and  $\gamma_{\alpha}$  is, in fact, the indicator of faultlessness of the calculation algorithm. There is virtually an insignificant divergence between correlation coefficients  $\gamma_{1,m,\alpha}$ ,  $\gamma_{2,m,\alpha}$  and  $\rho_{2,m,\alpha}$ . It makes up no more than 2,5%, which experimentally confirms the existing statements about equality of  $\gamma_{1,m,\alpha}$  and  $\rho_{2,m,\alpha}$ . However, the attempts to use these relations to state the equality between  $\gamma_3$  and  $\rho_{3,3}$  calculated using experimental data, can lead to incorrect conclusions as estimates can differ greatly.

An analysis of SFFD of correlation coefficients indicates that:

- at  $n_s=5 \div 10$  the function of the fiducial distribution of  $F^*(\rho_{1,m})$  has a pronounced discrete character;
- SFFD of  $F^*(\rho_{1,m})$  is symmetric with respect to  $\rho_{1,m}=0$  even at  $n_s=5$  and, therefore, can be represented as a distribution of absolute value  $\rho_{1,m}$ ;

Table 8. The relationship between sample size and  $\Delta\rho_M$ .

| $n_s$          | 5   | 6      | 7      | 9      | 9      | 10     |
|----------------|-----|--------|--------|--------|--------|--------|
| $\Delta\rho_M$ | 0,1 | 0,0571 | 0,0357 | 0,0238 | 0,0167 | 0,0121 |

- the sampling increment  $\Delta\rho_M$  of an argument of distribution of  $F^*(\rho_{1,M})$  depends on the value of  $n_s$ . The size of  $\Delta\rho_M$  decreases nonlinearly with an increase in  $n_s$ . Table 8 presents the relationship between  $n_s$  and  $\Delta\rho_M$ .

It is worth noting, that decision H is traditionally made according to criterion (2) developed for continuous random variables. In fact, there are no critical values  $\rho_{1,\alpha}$  corresponding to  $\alpha$  at discrete nature of change in  $F^*(\rho_{1,M})$ . Therefore, we propose comparing the probabilities of  $F^*(\rho_{1,M})$  and  $\alpha_k$  rather than quintiles of distribution of  $F^*(\rho_{1,M})$  (i.e.  $\rho_{op}$  and  $\rho_{M,\alpha}$ ):

$$\text{if } \rho_{1,M,j} \leq \rho_{op} < \rho_{1,M,(j+1)} \text{ and } F^*(\rho_{1,M}) < (1-\alpha_k),$$

then  $H \Rightarrow H_1$  else  $H \Rightarrow H_2$ .

Modeling results and quantiles of correlation coefficients at  $\alpha=0,5$  presented for illustrative purposes testify to their equality and identity of the change pattern for the specified volume of sampling  $n_s$ , SFFDs  $F^*(\gamma_1)$ ,  $F^*(\gamma_2)$ ,  $F^*(\rho_1)$  and  $F^*(\rho_2)$ . Distributions of correlation coefficients  $\gamma_{1,M}$  and  $\rho_{2,M}$  which are graphically presented in Fig.2, make it possible to conclude that the distribution of correlation coefficients both from  $\rho_1$ , and  $\rho_2$  has a discrete nature and influence of discreteness declines with an increase in  $n_s$ .

Discreteness of distribution of  $F^*(\rho_2)$  decreases owing to the consideration of identical values of sample realizations.

## V. CONCLUSIONS

The developed method for formation of a list of independent techno-economic indicators makes it possible to transition from a full list of the indicators to individual ones, which is an indispensable condition for increasing the objectivity of integrated estimates of overall performance of electric power system facilities. The proposed method is based on the following.

1. A method for transformation of a numerical scale of measurement of the techno-economic indicators to a rank-order one. This method allows increasing both the number of the attributes characterizing overall performance of the objects and reliability of the decision by means of transition to nonparametric criteria for the assessment of the relationship between the indices (parameters) of the attributes to be considered;

2. The possibility of the automated assessment of critical values of linear and rank correlation coefficients by constructing the statistical distribution function of the modeled realizations;

3. A possible essential distinction of experimental values of linear and ranking correlation coefficients at their almost identical critical values.

4. A method intended to take into account the differences in changes in techno-economic indicators.

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