Markov expert logical analysis for energy reserves probabilistic evaluation

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***Abstract* — The paper focuses on the probabilistic evaluation of oil and gas resources with the models and methods of AHP/ANP analysis. The AHP/ANP models are shown to be the particular cases of finite Markov chains, i.e. discrete random processes with Markov property. An integrated method (Markov expert logical analysis (MELA)) is proposed. The method is based on the models, methods and algorithms of Markov chains theory. This basis will stimulate the progress in research on multi-criteria decision-making problems that arise in various spheres. The paper presents different methods using MELA to allow for the uncertainty of numeric and nonnumeric data on gas reserves as methods of transformation of expert estimations into the probability distributions. Typical logical schemes are proposed for multi-criteria comparison of analogous objects, to take account of possible errors in porosity evaluation and to estimate project life.**

***Index Terms* — Analytic Hierarchy Process (AHP); Analytic Network Process (ANP); Markov chains; evaluation of resources; Markov expert logical analysis (MELA); logical scheme; information uncertainty; Monte-Carlo method; project life; oil and gas resources probabilistic evaluation.**

I. Introduction

Probabilistic evaluation of geological oil and gas resources is an important objective of regional energy research. The hydrocarbon (HC) resources are evaluated by experts (geologists, geophysicists) on the basis of their judgments about petrology and process of reserve formation; generation, migration and dissipation of HC-fluid and other geological information, usually of hypothetical nature. In this case, there can be significant discrepancies in judgments by different experts. As this information is usually qualitative, it is normal to use the methods of expert logical analysis (ELA) that provide experts with universal language for analysis and agreement of final estimates.

Two main ELA methods [1,2] – analytic hierarchy process (AHP) and analytic network process (ANP) – allow us to probabilistically evaluate the porosity and permeability parameters of oil and gas reservoirs, from which it is easy to probabilistically evaluate the volume of the resources. The values of expert probabilities reflect expert's confidence in the correctness of the parameters evaluation.

II. AHP and ANP

The AHP and ANP suggest special logical schemes to organize the evaluation procedure (Figure 1).

The logical scheme of AHP (hierarchy) is characterized by the following special aspects:

all its elements are grouped in T+1 classes (levels of hierarchy) S_t, $t = 0,1,\ldots,T$, so that in class S_0 one element 0 is included, showing the aim of investigation, in class S_T elements correspond to variants of decision-making ("alternatives"), elements of the other classes can have a certain meaning (actors, groups of criteria, criteria, factors and others);

links (indicated by arrows) exist only between the elements of neighboring levels. This means that elements of one level must be independent and they cannot influence each other.

In the models of analytical network these requirements are withdrawn. This means that logical schemes of ANP can be optional.

The second part of AHP and ANP consists of the methods of giving priorities (weight) $p_{ij}(t)$ to elements $i \in S_t$, $t = 0,1,\ldots,T$, $\sum_j p_{ij} = 1$, which are stated in [2,3]

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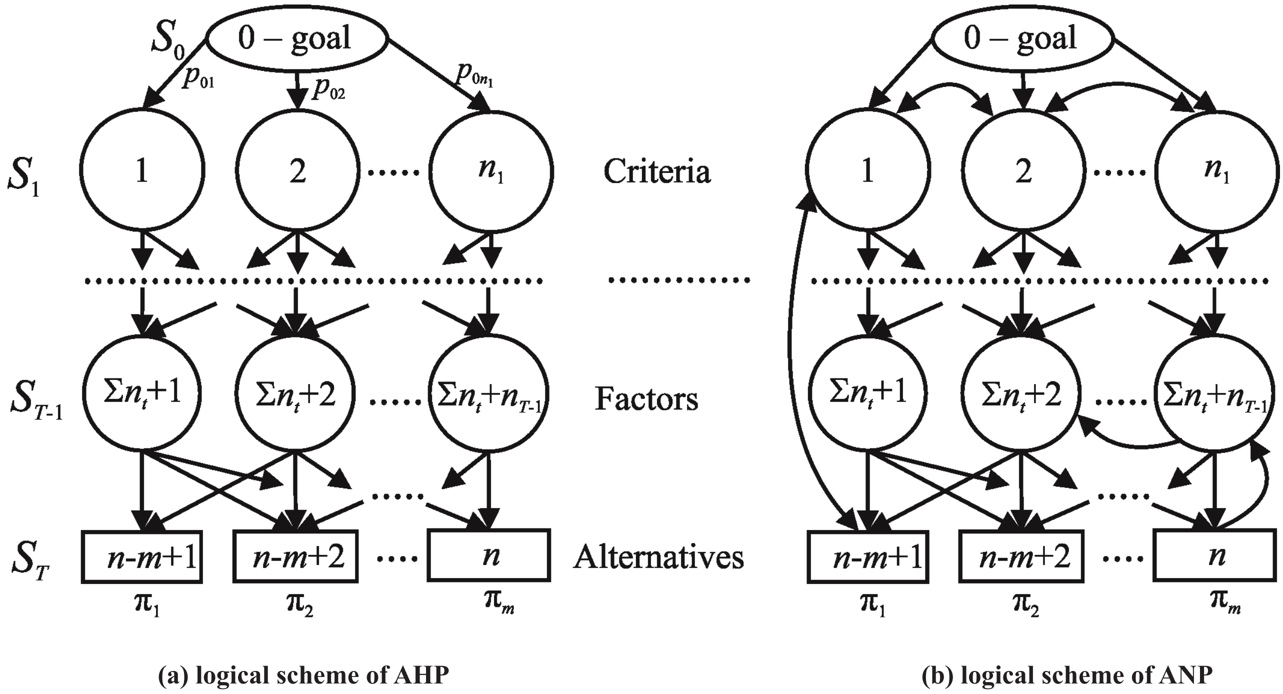


Fig. 1. A general layout of logical schemes of AHP (a) and ANP (b).

These methods allow comparing the measured indicators as well as qualitative ones for calculating quantitative evaluations of preferable decisions (absolute priorities, weight).

The third part of AHP and ANP consists of calculating absolute priorities π_k of alternatives $k = 1,\ldots,m$ with logical scheme and of relative priorities $\{p_{ij}\}$, $i = 0,1,\ldots,n$.

Let us enumerate some important advantages of AHP:

- It makes it possible to effectively breakdown complicated schemes of analysis, which create favorable possibilities for dividing the complicated problems into a set of simple ones and combining their decisions;
- It controls logical conformity of expert's judgment, which allows avoiding logical errors in expert evaluations;
- It provides evaluation of priorities, resistant to small data variations.

The results of investigations with the AHP and ANP schemes (absolute priorities π_k of alternative $k = 1,\ldots,m$) are convenient to interpret as a share of total amount of votes during election, determined by logical scheme. In the decision making tasks, this allows ranging alternatives in correspondence with their importance. Another important interpretation lies in that π_k are expertprobabilities of accidental falling out of relative alternatives.

Neglect of the connections between elements can essentially influence the assessments of priorities. Therefore, the use of AHP, for example, under depending (correlated) criteria can lead to corrupted evaluations of priorities of alternatives. In this case, it is necessary to switch from AHP to the method of analytical networks, by reflecting all links in the logical scheme. In practice, this limits greatly the applicability of AHP.

III. Markov expert logical analysis

The authors of [3] state that any logical scheme of ANP after simple transformations (markovization) is isomorphic to the transition graph of some homogeneous Markov chain (MC) with unlimited "time". This isomorphism is assigned by the analogy of the main concepts of analytic networks (AN) [2, 3] and MC (Table 1).

Markov expert logical analysis (MELA) is an evident generalization of ANP, which requires fulfillment of the following stages:

1. Prepare logical scheme for considering the problem as a transition graph of Markov chain;
2. Choose the methods for evaluating relative priorities equivalent to probabilities of transitions between the MC states and form the transition matrix of probabilities;
3. Markovize analytical network by adding fictitious vertices and edges to AN so that the limit probabilities (or average limit probabilities for periodic chain) coincide with the distribution of ANP absolute priorities of alternatives;
4. Assign the initial values $p(0)$ of probabilities to all MC states;
5. Insert the data in the program and calculate the limit probabilities for all states of non-periodic MC (or the average probabilities of MC states for periodic one) that are considered as absolute priorities of logical

scheme of elements;

6. Control an agreement of expert judgments (correct them if some of them do not agree and repeat the calculations);
7. Interpret elements of decision in accordance with the sense of problem and correction of logical scheme, probabilities of transition and reevaluating relative and absolute priorities if necessary.

For the analytical network markovization, it is normally enough to make the following transformations of its graph G (Figure 2):

a. Build the condensation of graph G [4], point out all connected components in it, source components and sink components (Figure 2b);

b. Add fictitious vertex 0 (“common source”), fictitious vertex Z (“common sink”) and fictitious edges connecting sinks with sources in graph G. Assign probabilities to edges, defined by the following simple rules:

Assign equal transient probabilities p_{0j} (equal to 1 in sum) to the edges (0, j) coming into source blocks;

If i is the only vertex of sink block, assign transition probability $p_{iZ} = 1$ to edge (i, Z). In the event that the sink block contains several vertices, assign small probability $p_{iZ} = \delta$ to edge (i, Z), and divide probabilities p_{ij} of the other transitions from vertex i by $1 + \delta$;

- Assign the probability of transition $p_{Z0} = 1$ to the fictitious edge (Z, 0).

It is obvious that such markovization makes the connected graph of analytical network strongly connected. This algorithm can be easily automated and the program will do all these operations by itself.

An analysis of the AN structure with the methods of the theory of graphs helps to reveal logical discrepancy and in some cases to significantly simplify the AN.

IV. Problems of hydrocarbon resources evaluation

The above-described method can be applied to probabilistic evaluation of initial V and extracted V_{ex} volumes of hydrocarbon (HC) resources. The main difficulties of such an evaluation are connected with the lack of reliable data on the shape and volume of the pore space Ω of reservoir and its properties: porosity m, gas α or oil α_o saturation, reservoir fluid composition, as well as pressure p and temperature T in the reservoir.

Let us consider gas reserves. Geometrical form of the pore space Ω, gas content and parameters α, m, p, T are not known precisely. In fact, they are assigned by geologists who base their estimations on the results of geophysical investigations and the data on lithological characteristics of reservoirs and the processes of hydrocarbon formation, migration, accumulation and dissipation in it.

It is common practice to substitute a reservoir with a homogeneous isotropic cylinder of the same volume with horizontal sole with an area S, taking $\Omega = hS\cos\varphi$, where h – average thickness of reservoir, φ – formation dip. For the volume of initial V and extracted V_{ex} gas reserves, the following formula are known:

$$V = C\alpha mhS\cos\varphi\frac{PT_0Z_0}{P_0TZ};\ V_{ex} \approx V\eta, \tag{1}$$

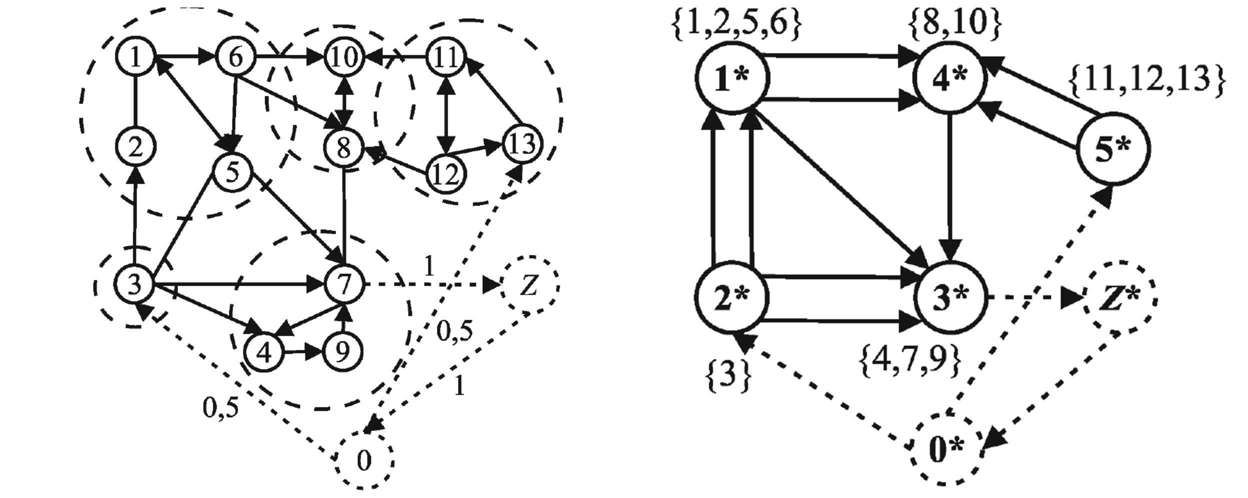
where η – average gas-recovery factor, P – average reservoir pressure, $Z = Z(P,T)$ – gas-compressibility factor (depends on gas composition), coefficient C takes into account units of measurements, and index 0 indicates standard value of magnitudes. It is reasonable to consider parameters α, m, h, S, P and η as random variables.

MELA can be used to take into account the uncertainty of both numeric and nonnumeric data on gas formation as a method for transformation of experts’ estimations into the probability distribution series of evaluated magnitudes. We will consider two ways of building such distributions:

1. For insufficiently explored objects, this is the use of MELA to set discrete vector distributions $\{\pi_j, X_j\}$ (Figure 3). This method is based on comparison of a considered object with a set from n analog objects (“alternatives” in the MELA scheme). Here π_j is the probability that $X = X_j$ (weight of the j-th alternative), and $X_j = (x_{jk})$ is vector

Table 1. Analogy between AHP-ANP and Markov chains terms

Terms of T.Saati [1,2]	Standard terms of MC [4,5]
AN, logical scheme	Transition graph of MC
Element of analytical network	State of Markov chain
Component of analytical network	Subset of states of MC
Alternatives	States with numbers n-m+1,…, n
Influence of element i on element j	Transition from state i to state j
Relative priority of influence of element i on element j	Probability of transition from state i to state j
Absolute priority of element j	Limit probability of state j
Super-matrix of analytical network	Matrix of transition probabilities
Vector w of absolute priorities of the analytical network elements	Vector π of limit probabilities (or their Cesaro averages)
Component of structural graph	Component of connection
Source components	Subset of non-recurrent states
Sink components	Classes of adsorbing states
Structural graph	Graph condensation



a. Graph G of analytic network and markovization b. Condensation of graph G and markovization

Figure 2. Markovization of analytic network

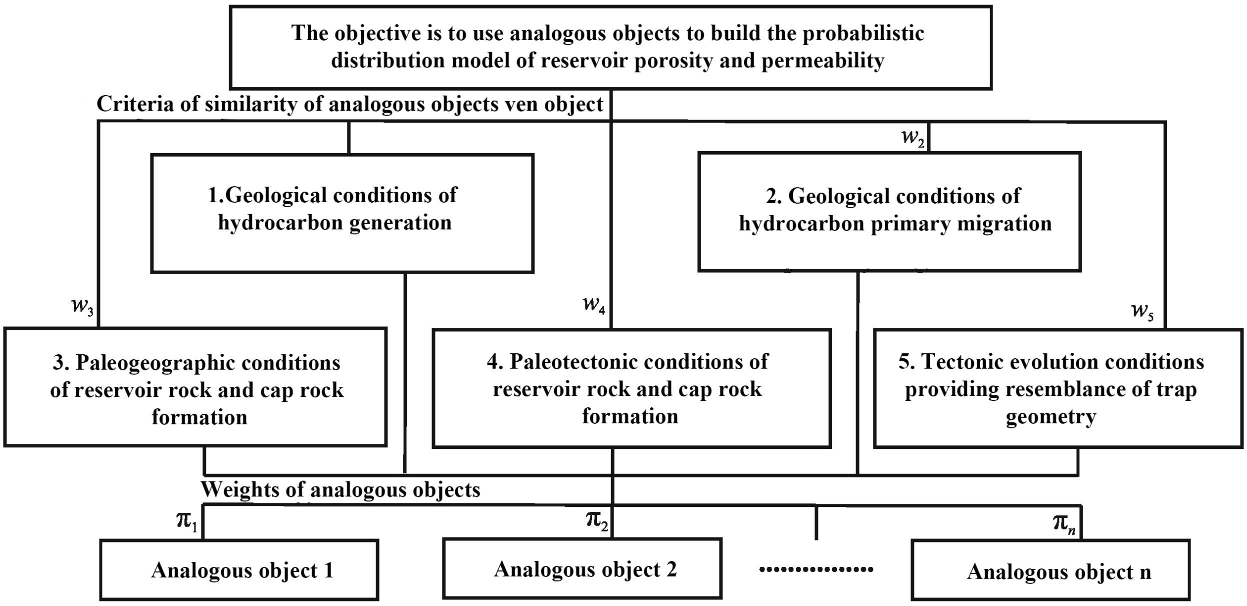


Figure 3. Logical scheme of multi-criteria comparison of analog objects to construct a probabilistic distribution model of reservoir parameters of the insufficiently explored object in question

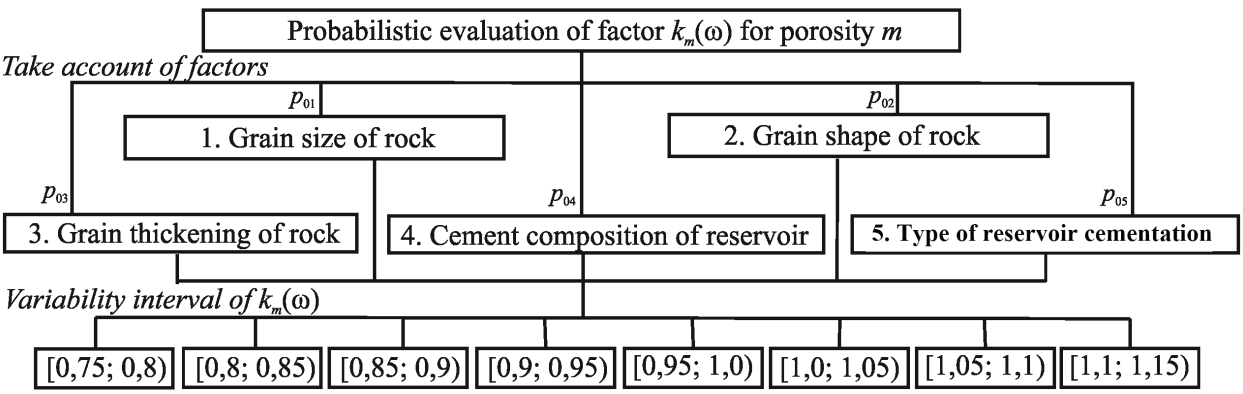


Figure 4. Typical MELA scheme for evaluation of factor km(ω) to allow for possible errors in porosity evaluation m (p_{0k} – weights of criteria)

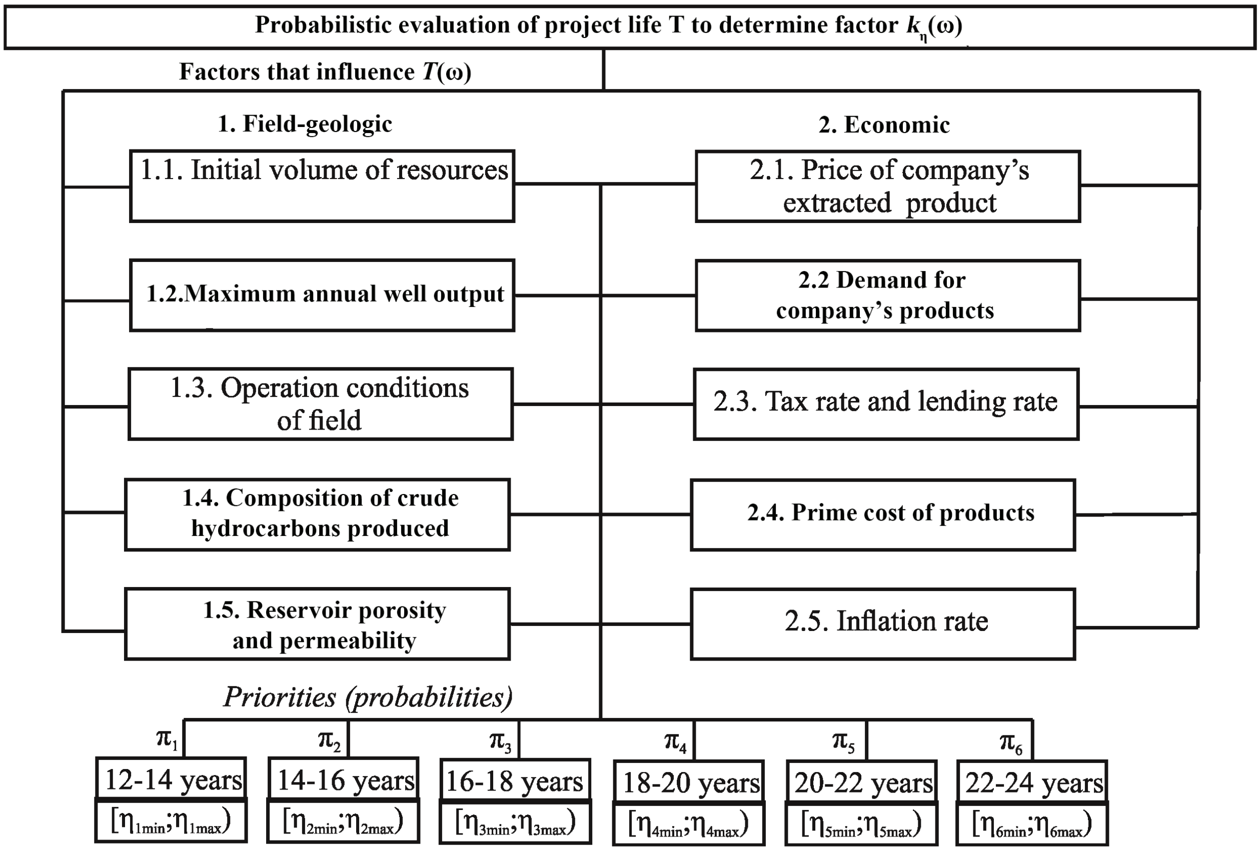


Figure 5. A typical MELA scheme for evaluation of project life and factor $k_\eta(\omega)$

of parameters of the j-th analog object, $j = 1,\ldots,n$. It is recommended to widen the list of analog objects;

2. For more studied objects, the MELA scheme is used to set the distribution $\{\pi_j, x_j\}$ of probability that parameter x belongs to the given intervals: where π_j – probability; x_j – average value of the *j-th* interval, $j = 1,\ldots,n$.

The logical scheme of AHP can be used to evaluate the proximity measure of analog objects to the insufficiently studied object (Figure 3). The weight coefficients w_i assess the importance of criteria indicating the proximity measures of formation conditions of a given object and analog objects. The closer the considered field to object j in terms of formation conditions, the larger the weight π_j of this analog object.

Figure 3. Logical scheme of multi-criteria comparison of analog objects to construct a probabilistic distribution model of reservoir parameters of the insufficiently explored object in question

The weights can be considered as expert assessments of probability of coincidence of porosity and permeability parameters of an object with those of an analogous object. The schemes of logic assessment can serve as data sources for building the probabilistic models of the object characteristics. The results of the evaluation specify the probabilities to vectors of parameters α, m, h, S, P :

Probability (weight), π_j: $\pi_1, \pi_2, \ldots, \pi_n$; Value of porosity and permeability parameters - vector dimension,

$$X_j: X_1 = (x_{1k}), X_2 = (x_{2k}), \ldots, X_n = (x_{nk}).$$

Thus, the unknown vector X of porosity and permeability parameters of this gas field can be predicted as the mean value by the formula:

$$X \approx \sum_{j=1}^{n} \pi_j X_j .$$

The second method suggests the following actions:

expert chooses the parameters in (1) to be considered as random variables. For each of them, we introduce a correction factor (random variable) $k_{.}(\omega)$ with corresponding index for the base value of the evaluated parameter $k(\omega)=1$ (ω – vector of random factors);

expert uses a typical MELA scheme or makes a logical MELA scheme for each factor $k(\omega)$ in (1), where alternatives are represented by sub-intervals of an interval of possible values $k(\omega)$;

MELA is used to calculate the empirical distribution $\{\pi_j, k_j\}$ of probabilities of factor $k(\omega)$ (k_j being the middle point of the *j-th* sub-interval);

factors $k(\omega)$ are assumed to be independent random variables, based on their empirical distributions the representative sample $\{V(\omega)\}$ of possible volume of reserves is generated with the Monte-Carlo method

[7]; the sample $\{V(\omega)\}$ is processed by the methods of nonparametric statistics [5].

Probabilistic evaluation of gas reserves in the field is determined by the formula:

$$V_{ex}(\omega) \approx V_{ex} k_{\alpha}(\omega)\, k_m(\omega)\, k_h(\omega)\, k_S(\omega)\, k_p(\omega)\, k_{\eta}(\omega) = k_V(\omega)\, V, \quad (2)$$

where $k_V(\omega)$ is a random factor for evaluating the volume of extracted gas reserves. Variable $k(\omega)$–1 indicates the value of a random error in the parameter with respect to its base value.

Fig. 4 demonstrates the MELA scheme for evaluation of $k_m(\omega)$. Expert has the right to correct it, add (delete) elements and links. Logical schemes for parameters $k_{\alpha}(\omega)$, $k_h(\omega)$, $k_S(\omega)$, $k_p(\omega)$, $k_{\eta}(\omega)$, included in $k_V(\omega)$, are built in the same way.

Fig. 4. Typical MELA scheme for evaluation of factor $k_m(\omega)$ to allow for possible errors in porosity evaluation m (p_{0k} – weights of criteria)

Correction $k_{\eta}(\omega)$ to the gas recovery factor (GRF) depends not only on geologic factors (characteristics of reservoirs, entrapment of gas with fallen condensate, etc.) but also on economic-geographical ones. The former factors are determined by conditions of formation of the reserves and geological characteristics of this region. The latter group can be divided into engineering-technological and economic factors requiring substantiation of GRF as a solution to the engineering-technological problem of determining the field development time. The first sub-group determines engineering-technological solutions for drilling and creation of a system for gas collection and preparation for transportation that should function during the whole life cycle of the field. The second sub-group is connected with the hypotheses about economic standard (unit costs of equipment and construction and installation work, operating costs, prices per unit, tax and lending rates, etc.).

The main factors of the first sub-group are the production horizon characteristics connected with the considered field (accumulation): type of reservoir, inhomogeneity and variability of massive material, tectonic features, deformation properties of massive material, type of accumulation, gas column, occurrence depth, characteristics of productive strata penetration, initial thermobaric conditions, recovery mechanism, reservoir gas content.

The gas recovery factor can be correctly evaluated only in the last stage of development. At the beginning of the development only its approximate evaluation is possible. The evaluation requires special stochastic optimization models with a criterion of maximum mean net present value [6]. Significant part of capital investment falls on reconstruction of the system for gas collection and preparation for transportation. These evaluations in the stage of decision making about the field exploration are approximate and require at least a simplified probabilistic risk analysis.

The GRF evaluation based on the materials of exploration drilling is made for the approval of reserves by State Commission of natural resources. The aim is to reveal the hydrocarbon volumes with a view to estimating capital investment in the system of production, transportation, processing, product distribution and determination of taxation basis. Normally, gas-dynamic calculations and GRF evaluation lack information, consequently, statistic data, analogies and expert evaluations are often used.

This forces the use of approximate evaluations by MELA based either on statistic data, or data from analogous objects, or judgments of geologists and economists. First two approaches are quite obvious (similar to the last examples) but they are usually not provided with necessary information. The third approach corresponds to the practice of GRF evaluation. Its scheme is presented in Figure 5.

The recovery factor is strongly correlated with the life of a project for exploration of a geological object, which should also be taken as random variable $T(\omega)$ (we consider here approximate evaluations of economic figures for geological objects of categories not higher than C_1). The above-listed factors (geologic-productive parameters, economic cost indicators and effects) influence the variable $T(\omega)$: factor $k_V(\omega)$ is evaluated by the Monte-Carlo method [7], which involves processing of a generated sample by the methods of nonparametric statistics [5].

The use of logical scheme does not reject the traditional calculations of supposed dynamics of techno-economic indicators of the project.

V. Conclusion

The evaluations of oil and gas resources in the early stages of choosing an object for exploration are based on geologists' concepts about structure and parameters of the fields, which is associated with high uncertainty and non-objectiveness of the evaluations. Generalization and application of modern methods for expert analysis of data and probabilistic models of planning exploration work allow us to numerically evaluate the influence of these factors on the field development indicators.

The evaluations made by geologists are based on detailed structural maps as well as on the notions of reserve formation history. It is necessary to connect, generalize and reconcile these notions with geophysical and field data. This is qualitative information and therefore AHP and ANP are recommended for evaluation, because they provide experts with universal language for analysis and agreement of their judgments.

AHP imposes strict restrictions on selection of a logical scheme and requires independence of elements of each level of hierarchy. Failure to meet these requirements may lead to significant errors in results. ANP is free of these restrictions but has no clear theoretical framework and needs a strict proof of computational algorithm.

The study indicates that AHP and ANP are special cases of homogeneous Markov chains, if we use the methods of markovization of the AHP/ANP logical schemes. This makes it possible to use Markov chains for analysis of logical schemes. The generalizations made constitute a theoretical basis for a new method of Markov expert logical analysis (MELA). This method enables multi-criteria decision making and multifactor analysis of data, by applying standard methods of Markov chains analysis.

Markov expert logical analysis, in particular, can be applied in probabilistic evaluation of resources in hydrocarbon fields. It determines the probability distributions for parameters of oil and gas reservoirs and recovery factor. The special schemes of MELA are recommended to find out the distribution series for resources, by evaluating similarities between the considered object and analogous objects. Thus, the probabilistic expert evaluation of parameters becomes a regulated procedure with formally controlled results.

References

[1] Saaty T. L. “The Analytic Hierarchy Process,” McGraw-Hill, New York. 1980.

[2] Saaty T.L., Vargas L. G. “Decision Making with the Analytic Network Process,” Springer, 1996.

[3] Saaty T. L., Özdemir, M. S. “The Encyclion: A Dictionary of Decisions with Dependence and Feedback based on the Analytic Network Process,” RWS Publications, USA. 2005.

[4] Stavrovsky E. “AHP and ANP as Particular Cases of Markov Chains,” E. Stavrovsky, M. Krasilnikova, S. Pryadko // IFAC Conference on Manufacturing Modeling, Management, and Control. Saint Petersburg, 2013, v.7, Part 1, pp. 531-536.

[5] Berzh K. “Theory of graphs and its applications,” Moscow: Izdatelstvo inostrannoj litereturi, 1962. 319 p. (in Russian).

[6] Orlov A. I. “Practical statistics. M.: Ekzamen,” 2004. 656 p. (in Russian).

[7] Stavrovsky E. Model of multistep stochastic design of systems “Short-cut gas field assets – gas line – consumer,” Stavrovsky E., Fomin E., Shmyglya O. // Pipeline systems of energetics. Management of development and functioning. Novosibirsk: Nauka, 2004. P. 135-145. (in Russian).

[8] Sobol I. “Numerical methods of Monte-Carlo,” Moscow: Nauka, 1973. 312 p. (in Russian).

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