

# Reliability Assessment of Electric Power System with Distributed Generation Facilities

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**Abstract** — Modern power supply systems that have distributed generation and are connected to the electric power system, renewable energy sources, and storage devices, require changes in the assessment of their reliability indices. The complexity of the energy, technological, and organizational structures of power systems with distributed generation does not allow the traditional concept of "failure" to be used to assess their reliability. Many technological solutions used in the distributed generation projects can become sources of vulnerabilities in the infrastructure of an intelligent electrical network. The study shows that power systems with distributed generation are the structures with overlapping service areas, which determines their specific features represented by an integral characteristic - efficiency. It characterizes the extent to which the use of distributed generation facilities in various operating conditions is feasible. The paper proposes an approach to quantifying the efficiency of such systems. The presented examples demonstrate the calculation of relatively simple power systems with the distributed generation that perform several tasks simultaneously.

**Index Terms** — distributed generation, electrical system, efficiency, reliability, power supply, structure

## I. INTRODUCTION

A specific feature of power systems with distributed generation (DG) is the complexity of their energy, technological, and organizational structure, which allows, on the one hand, performing a set of tasks to ensure reliable and efficient power supply to consumers, and on the other

hand, ensuring stable operation when the individual generating units (GUs) and their groups fail. Changes in the structure of the DG-based power system due to failures of groups of units and tie lines between them only decrease its performance indices, since the boundaries between operating and non-operating, as well as between operating, partially operating and non-operating states, are blurred and often conventional [1].

This is due to the redundancy of the structure, the presence of back-up generating units (overlapping service areas), switching capabilities and connections, specific features of the operation of relay protection devices and emergency control systems, operation correction tools (reactive power compensation and voltage regulation), and possible errors of the staff. Therefore, there is no universally accepted concept of "failure" for such systems [1, 2]. An example of overlapping service areas can be the case when several companies have distributed generators that are connected to the public power supply system.

The representation of DG-based power systems by structures with several overlapping service areas determines the specific features of their operation. These can be estimated by an integral characteristic - the efficiency, i.e., the extent to which the use of the DG-based power system in various operating conditions is feasible. However, the issues related to the specific features of calculating the efficiency of the systems with overlapping service areas have not been fully studied [2–7], which determines the significance of the research and the need for the practical implementation of the method of overlapping areas.

Even in a state of full operability, the DG-based power system may fail to perform all its functions, which can be due to an unfavorable combination of circumstances (uneven power generation by renewable energy sources (RES), overload due to increased consumer demand, and unauthorized external impacts (cyberattacks)) [8].

It is worth noting that the behavior of autonomously working generating units in the power systems with distributed generation differs significantly from that within the system. In the latter case, the specific features of the hierarchical structure, operating conditions, and switching capabilities of the network; the interdependence of failures of individual components; the order of their recoveries, etc. can manifest themselves. This necessitates taking

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into account the behavior of the  $k$ -th component ( $x_k$ ), which is determined by the prehistory of part or all other components [4]:

$$x_k(t+dt) = f_k[x_k(0,t), x_1(0,t), \dots, x_n(0,t)],$$

Where  $f_k$  is the function determined by the specific features of the system.

Then, the state of the system is determined by an  $n$ -dimensional random variable

$$X(t) = [x_1(t), x_2(t), \dots, x_n(t)],$$

whose path of behavior in the interval  $(t, t+\theta)$  is an  $n$ -dimensional random function

$$X(t,\theta) = [x_1(t,\theta), x_2(t,\theta), \dots, x_n(t,\theta)].$$

Therefore, it becomes necessary to expand the concept of “reliability” [9] and transition to the efficiency indices of a system with the distributed generation when it performs certain tasks in specific conditions [2–5]. The index of efficiency is a measure that quantitatively assesses the quality of how the DG-based power system fulfills the functions, that is, the measure of the utility of the DG-based power system functioning in a particular situation while supplying power to consumers under specific operating conditions of the generating units. In fact, the use of efficiency indices in the evaluation of the quality (extent) of performing their functions by complex electric power systems is a rather widely used approach.

It is worth noting that some of the reliability indices of complex power systems, for example, when assessing the adequacy, have a sense of efficiency indices (for example a mean value of undersupplied power).

## II. OPERATING CONDITIONS OF RADIAL POWER SUPPLY SYSTEM

1) By analogy with [10], we consider the simplest (autonomous) radial power supply system consisting of a conventionally failure-free source  $I$  (infinite bus), homogeneous consumers  $m$ , and identical power lines  $w$  (Fig. 1). We will define by  $s = 1-r$  the probability of a failure of the  $m_i$ -th consumer for the reasons that are not related to its power supply and by  $q = 1-p$  a failure of the power line.

Since the failures of the consumer and power lines are independent, we conclude that the probability that consumer  $i$  is connected to the source is  $P = rp$ . The probability of a complementary event is.  $Q = 1-rp$

Here, the distribution of the number of consumers connected to the source obeys the binomial law for which the mathematical expectation  $M[m]$  and standard deviation are defined by

$$M[m] = mrp; \quad \sigma_m = \sqrt{D[m]} = \sqrt{mrp(1-rp)}.$$

2) The development of renewable energy sources and distributed generation systems causes the need to estimate the operating conditions of autonomous electrical systems consisting of several sources. Consider a scheme of an autonomous electric power system (Fig. 2) for which it

is necessary to evaluate the reliability of power supply to essential consumers connected to the switchboard SB [11].

The structural scheme of this electric power system in the form of a graph (excluding circuit breakers) is shown in Fig. 3. Consumers receive power (operability of this power system) if the following conditions for normal operation are met: a) properly operating generator ( $G_1$ ), main switchboard board ( $MSB_1$ ), cable line  $W_1$ , switchboard (SB); b) properly operating  $G_1$ ,  $MSB_1$ ,  $W_3$ ,  $MSB_2$ ,  $W_2$ , SB; c) properly operating  $G_2$ ,  $MSB_2$ ,  $W_2$ , SB; d) properly operating  $G_2$ ,  $MSB_2$ ,  $W_3$ ,  $MSB_1$ ,  $W_1$ , SB.

$$A(v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8) = \begin{vmatrix} P_1 \\ P_2 \\ P \\ P_4 \end{vmatrix} = \begin{vmatrix} v_1 & v_3 & v_5 & v_7 \\ v_1 & v_3 & v_8 & v_4 & v_6 & v_7 \\ v_2 & v_4 & v_6 & v_7 \\ v_2 & v_4 & v_8 & v_3 & v_5 & v_7 \end{vmatrix}$$

The same conditions will be written in a matrix form

3) The expansion of closed networks in DG-based

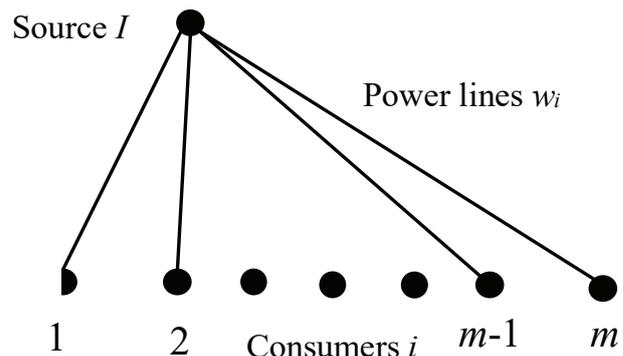


Fig. 1. Radial power supply system with a power source.

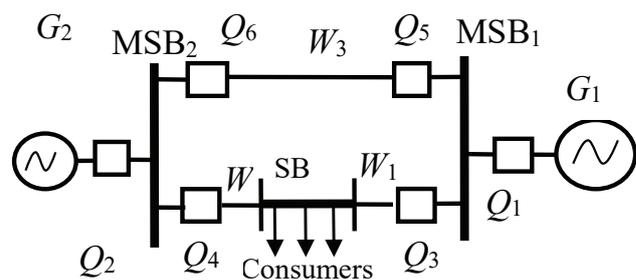


Fig. 2. Scheme of an autonomous electric power system.

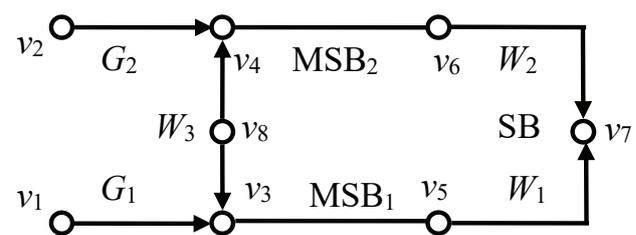


Fig. 3 Graph of the scheme in Fig. 2.

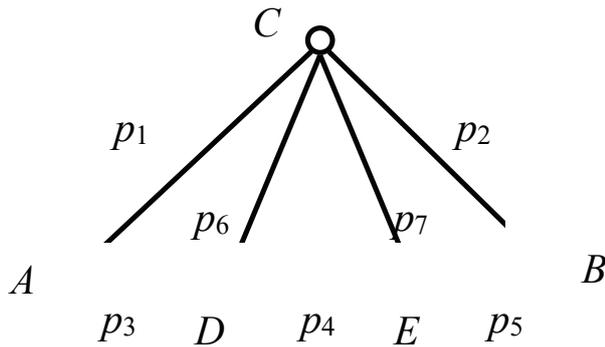


Fig. 4. Scheme of a conventional autonomous power system.



Fig. 5. A possible relationship between operating conditions of generating unit.

power systems requires an assessment of the probability of network connectivity. Figure 4 presents a graph of a similar system. The nodes of electricity generation, distribution, or consumption are the vertices of this graph, and the parameters of power transmission lines are the weights of edges that are equal to the probabilities of their operable state. In this statement, it is of interest to estimate the probability of network connectivity, for example, between nodes A and B. For simplification, all nodes are assumed to be reliable [10].

Suppose that for the tie line between nodes A and B, one can use all paths consisting of three or fewer edges connected in series. Then

$$\begin{matrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{matrix} \left\| \begin{matrix} AC & CB & 0 \\ AD & DE & EB \\ AD & DC & CB \\ AC & CE & EB \end{matrix} \right\|$$

The lack of the connection between nodes A and B is reduced to the calculation of the probability

$$Q_{AB} = (1-p_1 p_2) \cdot (1-p_3 p_4 p_5) \cdot (1-p_3 p_6 p_2) \cdot (1-p_1 p_7 p_5).$$

### III. DG-BASED POWER SYSTEMS WITH OVERLAPPING SERVICE AREAS

since any electric power system (EPS), including that with DG, belongs to the class of large systems with several levels of operation and is characterized by a complex structure, multi-functionality, and redundancy, it cannot be represented by single classical reliability indices. Failures of its components (in this case, generating units and tie lines between them) do not lead, as a rule, to the failure of the entire system but only reduce its efficiency.

In the simplest case [6], the efficiency is the probability that an autonomous generating unit will provide uninterrupted power supply to consumers. In this case, the efficiency coefficient  $E$  additionally acquires the meaning of probability that the performance of this task will not be disrupted due to the impossibility of providing some operating conditions (failures). We consider  $N_\Sigma$  to be a set of all possible operating conditions of generating unit aimed at providing power supply to consumers.  $N$  is a set of actually existing operating conditions. However, there can be some additional feasible conditions that together with  $N$  constitute a set of  $N_0$ . Then  $N \in N_0 \in N_\Sigma$  (Fig. 5).

The efficiency here is the probability  $p$  that the operating condition  $n$  belongs to the set  $N$ , that is,  $E = p\{n \in N\}$ . Similarly,  $E_0 = p\{n \in N_0\}$ . The probability that the operating condition of generating unit simultaneously belongs to two sets  $N$  and  $N_0$  will be

$$p\{n \in N, n \in N_0\} = p\{n \in N\},$$

because  $N \subset N_0$  (Fig. 1).

According to the multiplication theorem of probability, we obtain

$$p\{n \in N, n \in N_0\} = p\{n \in N_0\} \cdot p\{n \in N | n \in N_0\}.$$

In this case, the efficiency coefficient is determined as

$$E = \frac{p\{n \in N\}}{p\{n \in N_0\}} = \frac{p\{n \in N, n \in N_0\}}{p\{n \in N_0\}} = p\{n \in N | n \in N_0\}. \quad (1)$$

The resulting expression (1) is the probability that power will be supplied to consumers in almost any operating condition of the generating unit.

Since it is difficult to obtain the estimates of the efficiency of the DG-based power systems, the need arises to develop and improve the appropriate mathematical apparatus. In general, the estimation of the DG-based power system efficiency  $E_{syst}(t)$  is reduced to calculation using the formula proposed in [2]

$$E_{syst}(t) = \sum_{k=1}^n p_{X_k}(t) E_{X_k},$$

where  $X = (x_1, x_2, \dots, x_n)$  is a DG-based power system state;  $n$  is the number of the system's components in two states ( $x_i = 1$  – operable,  $x_i = 0$  – failure);  $p_{X_k}(t)$  is the probability that the system is in state  $X_k$  at time  $t$ ;  $E_{X_k}$  is the efficiency of the system in state  $X_k$ .

When  $n$  is large, the efficiency calculation using (2) is quite complicated. Therefore, we consider the approach proposed in [2, 3, 5]. It is based on the fact that in an area, there are several generating units integrated into a system. They supply power to consumers not only to those directly connected to «their» generating unit but also (with a sufficient power reserve and transfer capability of the network) to the consumers connected to other units that are geographically close and located in the overlapping areas of adjacent generating units.

If in the system of  $n$  components with probability  $p_i$

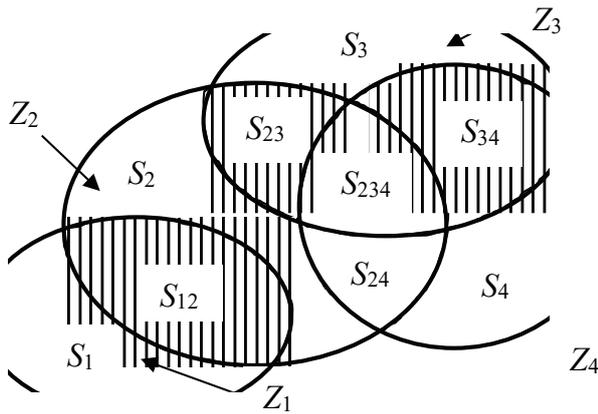


Fig. 6. Operating areas of generating units of the DG-based power system.

the  $i$ -th component is in an operable state, its service area includes area  $S_i$ . Service areas of two, three, etc. components can overlap and form areas  $S_{i,j}, S_{i,j,k}, \dots, S_{i,j,k,\dots,n}$  with corresponding components of influence. In the general case, there can be  $2^n$  such areas. The service area of the whole system is represented by the integration of all service areas of components  $S = \bigcup_{i=1}^n S_i$  [2].

Suppose that a conventional power system represents a DG-based power system that consists of four generating units (power plants), each providing operation of electricity consumers of «its» service area. There are regions in each area, which overlap with adjacent two or more areas. Consequently, a system with overlapping service areas is formed.

Part of the area that does not overlap with others is denoted by  $S_i, (i = \overline{1,4})$ , and the regions of overlap are  $S_{i,j}, S_{i,j,k} (i = j = k = \overline{1,4})$ . All highlighted regions define the service area  $S$  of a generating unit of the considered DG-based power system (Fig. 6).

Denote the efficiency coefficients of the highlighted regions by  $E$  with corresponding subscripts. If, for example, a generating unit whose main consumers are located in area  $Z_2$  fails, the efficiency coefficients of the regions covered by this area will change. Instead of  $E_{12}, E_{23}, E_{24}, E_{234}$  we obtain  $E_1, E_3, E_4, E_{34}$ . Naturally, in each practical task, the efficiency coefficients for the regions of overlap are determined individually.

Such a division of operating area of the DG-based power system consisting of  $n$  generating units can significantly reduce the number of the considered regions relative to the value of  $2^n$ , that is, it can simplify the calculation of  $E$  of the DG-based power system. The calculation of  $E$  is reduced to the calculation of the efficiency coefficients of each region and their weighted summing up given:

- power output from generating units;
- switching capabilities of power supply schemes;
- network operating parameters;
- composition, specific features of operation and settings of relay protection and emergency control systems;

- power consumption and process-related specific features of consumers located in the corresponding area;
- other indices characterizing the transition of the DG-based power system to limiting conditions.

The component of efficiency coefficient  $e_i$  in the resulting sum for area  $S_i$  without overlap  $S$  with other areas is found as

$$e_i = S_i E_i p_i,$$

where  $p_i$  is the probability of generating unit failure in area  $Z_i$  with the failure of power supply to consumers in area  $S_i$ .

The component for the region of overlap between areas  $Z_i$  and  $Z_j$  is equal to

$$e_{ij} = S_{ij} (E_{ij} p_i p_j + E_i p_i q_j + E_j q_i p_j)$$

The component formed by overlapping of three areas  $Z_i, Z_j$  and  $Z_k$  is

$$e_{ijk} = S_{ijk} (E_{ijk} p_i p_j p_k + E_{ij} p_i p_j q_k + E_{ik} p_i q_j p_k + E_{jk} q_i p_j p_k + E_i p_i q_j q_k + E_j q_i p_j q_k + E_k q_i q_j p_k)$$

The efficiency coefficients of each region in the case of a larger number of areas are estimated similarly.

Note that the efficiency coefficient determines the average level of quality of the system operation, which depends on the reliability of its components. It can be dimensionless (probability of meeting the consumer requirements) and can have the dimension (the amount of generated power, the amount of consumption, profit, loss, etc.) In the case of generating unit failure in area  $Z_i$ , the losses (damage) are determined by consequences of the interrupted power supply to consumers located in region  $S_i$ . In other regions of overlap with other areas, there will (or can) be some decrease in the efficiency (lower productivity, changes in the process parameters, and (or) the quality of products) of consumers located in these areas [1]. The damage  $Y_i$  in area  $S_i$  can be estimated by the expression

$$Y_i = S_i E_i + S_{ij} (E_{ij} - E_i) + S_{ijk} (E_{ijk} - E_i) + \dots$$

An approximate estimate of the magnitude of total damage  $Y_\Sigma$  based on the probability  $p_i$  of failures of generating units in different areas of DG-based power systems will be

$$Y_\Sigma = \sum_{i=1}^n p_i Y_i.$$

#### IV. ASSESSMENT OF EFFICIENCY OF DG-BASED POWER SYSTEMS WITH OVERLAPPING SERVICE AREAS

Consider a DG-based power system consisting of two identical generating units, each capable of operating in two conditions:  $x_1$  and  $x_2$ .

If both conditions  $x_1 \cup x_2$  are possible, then the DG-based power system efficiency is  $E = S_2$ ; if  $\bar{x}_1 \cup x_2$  or  $x_1 \cup \bar{x}_2$ , then  $E = S_1$ , (at  $\bar{x}_i, i = 1,2$  condition fails) and  $S_1 < S_2$ ; if  $\bar{x}_1 \cup \bar{x}_2$ , then  $E = 0$ [7].

The state of the DG-based power system is characterized by a vector with components

$$\{x_1^{(1)}x_2^{(1)}x_1^{(2)}x_2^{(2)}\}, \quad (3)$$

where the superscript is the generating unit number, and the subscript is a corresponding condition.

Each component (3) takes two values:  $x_i^j$  - power is supplied to consumers, and  $\bar{x}_i^j$  - power cannot be supplied.

When  $E = 0$  we have:

$$\{\bar{x}_1^{(1)}\bar{x}_2^{(1)}\bar{x}_1^{(2)}\bar{x}_2^{(2)}\}.$$

When  $E = S_1$ , there can be two conditions: *a*) arbitrary, but only one component of the vector (3) takes the value

$x_i^j$ , and others take the value  $\bar{x}_i^j$ ; *b*) the DG-based power system state is characterized by one of the vectors

$$\{\bar{x}_1^{(1)}x_2^{(1)}\bar{x}_1^{(2)}x_2^{(2)}\}, \{x_1^{(1)}\bar{x}_2^{(1)}x_1^{(2)}\bar{x}_2^{(2)}\}. \quad (4)$$

When  $E = S_2$ , there can also be two conditions: *a*) at least three components of the vector (3) take the value  $x_i^j$ , (one generating unit is serviceable); *b*) the DG-based power system state is characterized by one of the vectors

$$\{\bar{x}_1^{(1)}x_2^{(1)}x_1^{(2)}\bar{x}_2^{(2)}\}, \{x_1^{(1)}\bar{x}_2^{(1)}\bar{x}_1^{(2)}x_2^{(2)}\}. \quad (5)$$

Comparison of expressions (4) and (5) shows that in both cases the system consists of two generating units operating in one of the possible conditions; the number of impossible conditions is also the same, but efficiency indices are different. The difference is determined by the specific features of the power supply to consumers. According to (4), the DG-based power system appears to be fully operational at the expense of two partially operational generating units.

First of all, the efficiency assessment of the DG-based power systems should be based on fundamental knowledge about their structural, technological, and organizational features; operation principles of the main and control elements; relay protection systems; and automated systems, which constitute them [12, 13]. The order of even a preliminary assessment of the DG-based power system efficiency is as follows:

- obtain the information on the main scheme and operating characteristics and techno-economic parameters of the DG-based power system;
- find out possible tasks of DG-based power system operation;
- investigate the DG-based power system operating conditions in parallel with EPS, independently, and in a combined mode;
- estimate the expected frequency of repetition of tasks and operating conditions of the DG-based power system;
- build a functional diagram of the system;
- divide the DG-based power system into separate areas, and regions;
- select a quantitative measure of the DG-based power system operation quality, which is acceptable for this system;

- calculate the reliability indices of the components characterizing the probability of the state of each component at different time points;
- calculate the probabilities of average (if necessary feasible and/or limiting) states of the DG-based power system based on the probabilities of states of individual components;
- estimate the efficiency indices of possible states of the DG-based power system.
- If for each state of DG-based power system one can estimate an «instant» value of its efficiency (output effect), the efficiency coefficient is determined as an average value of the output effect for all states of the system [2].

## V. EXAMPLES OF A ROUGH ASSESSMENT OF THE DG-BASED POWER SYSTEM EFFICIENCY

### A. Non-overlapping service areas of generating units in the DG-based power system.

Each of the two identical generating units in the DG-based power system provides power supply to consumers in its area with probability  $p = 0.9$ . Assume the availability factor of each generating unit  $K_g = 0.95$ . For the system to be operational not only at any required time but also during a required interval  $t + t_0$ , we introduce an interval availability factor.

$$K_g^*(t, t + t_0).$$

In the general case,  $p^*(t) \neq p$ , since  $p$  is the probability of failure-free operation starting with the state of full operability, and  $p = 0.9$  is the probability of failure-free operation starting with one of the possible intermediate states. For simplification, we assume that  $t \rightarrow \infty$  and  $p^*(t) = p = 0.9$ .

Then

$$K_g^*(t) = K_g p^*(t).$$

In the case of a failure of one of the generating units, the DG-based power system splits into two independent subsystems, each capable of providing only half the load of consumers. The efficiency coefficient of one generating unit is determined as

$$e_{gi}(t) = 0.5K_g p = 0.5 \cdot 0.95 \cdot 0.9 = 0.4275.$$

Consequently, the considered DG-based power system provides the efficiency coefficient equal to  $E_\Sigma = 2e_{gi} = 0.855$ .

### B. Point estimation of the DG-based power system efficiency in the case of the simplest generating units.

The DG-based power supply system consists of two identical power plants (generating units) *Sa* and *Sb* that supply power to consumers with the maximum power consumption  $P_\Sigma = 36$  MW. The output of plant *Sa* varies from  $P = 0$  to  $P = 22$  MW, and that of plant *Sb* - from  $P = 14$  to  $P = 36$  MW. The power supply areas of these plants are shown in Fig. 7.

It is required to find the probability of supplying the load that occurs at any time in the range  $0 \leq P \leq 36$  MW if the reliability of supplying the necessary power from these

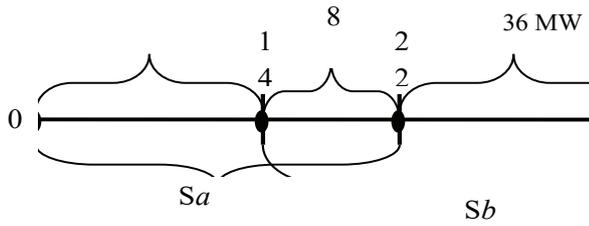


Fig. 7. Areas of power supply from the power plants.

plants at this time point is  $R_a = R_b = 0.95$ . At the same time, the probability of reliable power supply to the consumers located in the service area of each of the generating units is  $R_1 = R_2 = 0.9$ , and in the area of simultaneous service of both plants (in an overlapping area), it is

$$P = 1 - (1 - R_1)^2 = 0.99.$$

Suppose that the consumer load at a given time is  $P_0$  MW. The probability that the DG-based power system will be able to supply it at this time is

$$P_0 = R_a \cdot R_b = 0.95 \cdot 0.95 \approx 0.9.$$

At the interval from  $P = 14$  to  $P = 22$  MW (an overlapping area is  $P = 8$  MW), the consumer load is provided by both plants, while in the areas from  $P = 0$  to  $P = 14$  and from  $P = 22$  to  $P = 36$  MW – by one plant.

The efficiency index  $E_0$  of the DG-based power system state at a given load  $P_0$  MW is determined by the weighted average

$$E_0 = \frac{P_{S_a \cap S_b}}{P_\Sigma} P + \frac{P_{S_a - (S_a \cap S_b)} + P_{S_b - (S_a \cap S_b)}}{P_\Sigma} P_0$$

$$= \frac{8}{36} \cdot 0.99 + \frac{14 + 14}{36} \cdot 0.9 = 0.92.$$

Note that «weight» here is the characteristic of states of the DG-based power system, and not its separate components (generating units). In the same system, the «weight» of a component largely depends on the state for which it is considered. If all components operate but the  $i$ -th component fails, it is obvious that this failure is accompanied by consequences that differ from the case where this  $i$ -th component fails after the failure of one component  $j$  ( $i \neq j$ ) or several components of this system. However, for DG-based power systems, in many cases, a component that is more important in one state of the system proves to be more important in another state of the system as well.

The probability of the system state when one plant  $S_a$  operates within the load limits  $0 < P_a < 22$  MW and none of the plants works in the range  $22 < P < 36$  MW is

$$P_a = R_a (1 - R_a) = 0.95 \cdot 0.05 \approx 0.05$$

The efficiency index  $E_a$  of the DG-based power system state at a load of  $P_a$  MW will be determined as

$$E_a = \frac{22}{36} \cdot 0.9 + \frac{14}{36} \cdot 0 = 0.55.$$

Since in the given example, the states of  $S_a$  and  $S_b$  are identical, we obtain the resulting efficiency of the analyzed DG-based power system as follows:

$$E_\Sigma = P_0 E_0 + 2P_a E_a = 0.9 \cdot 0.92 + 2 \cdot 0.05 \cdot 0.55 = 0.883$$

C. Estimation of technical efficiency of the DG-based power system at a given time interval.

The DG-based power system consists of two identical power plants (generating units). If both plants are operational, the load provided by them is  $P$  MW. In the case where one of them fails, the value of the provided load is reduced to a value of  $0.3P$ . If two plants fail, the power supply to consumers is completely disrupted. The failures of the generating units are independent events. The probability of failure-free operation of each plant obeys the exponential distribution law

$$p(t) = e^{-\frac{t}{\tau}} = e^{-\lambda t}.$$

Considering the DG-based power systems of this type, we should take into account the fact that the generating units will not supply the required power to consumers during the shutdown because of preventive maintenance, or due to external factors (weather conditions, if generating units consist of renewable energy sources). However, to simplify the problem, we assume that the distribution remains exponential.

If we take the duration of the calculation period,  $\tau = t$  equal to the average failure-free operation period of each plant, i.e.,  $t = T = 1$  year, the probability of plant failure-free operation will be

$$p = e^{-\frac{t}{\tau}} = e^{-1} = 0.368.$$

Indices of technical efficiency of DG-based power system states are determined by multiplying the transmitted power by the time of operation:

technical efficiency of DG-based power system operation when both plants are operable during the considered period  $\tau$  is

$$E_0 = P\tau$$

technical efficiency of DG-based power system operation, when one of the generating units fails at time  $t_i$  is

$$E_i(t_i) = 0.3P\tau + 0.7Pt_i \quad t_i < \tau, \quad i = a, b;$$

technical efficiency of DG-based power system operation, when one generating unit fails at time  $t_i$ , and the other one – at time  $t_j$  is

$$E_{ij}(t_i, t_j) = 0.3Pt_j + 0.7Pt_i \quad t_i < t_j < \tau,$$

According to the theory presented in [4, 5], the resulting estimation of the efficiency of the analyzed DG-based power system is obtained by the formula:

$$E = p^2 P\tau + 2p\{0.3P\tau(1-p) + 0.7P\tau[1-p(1+\lambda\tau)]\} +$$

$$+ 2\{0.3P\tau[0.75-p(1+\lambda\tau)+0.25p^2(1+2\lambda\tau)] +$$

$$+ 0.7P\tau[0.25-p+(0.75+0.5\lambda\tau)p^2]\} =$$

$$= 0.368^2 P\tau + 2 \cdot 0.368\{0.3P\tau(1-0.368) +$$

$$+ 0.7P\tau[1-0.368(1+1)]\} +$$

$$+ 2\{0.3P\tau[0.75-0.368(1+1) + 0.25 \cdot 0.368^2(1+2)] +$$

$$+ 0.7P\tau[0.25 - 0.368 + (0.75+0.5) \cdot 0.368^2]\} =$$

$$= (0.135+0.28+0.138)P\tau = 0.553P\tau.$$

The analyzed DG-based power system can be viewed as a system with additive efficiency indices [4, 5] since each generating unit of this system contributes its independent

share  $c_k$  to the overall output effect. Then the efficiency of the system will be determined by the following expressions

$$\begin{aligned} E_0 &= P\tau\mathcal{P}_0 = P\tau[1 - (1 - p)^2] = \\ &= P\tau[1 - (1 - 0.368)^2] = 0.6P\tau; \\ E_i(t_i) &= 0.3P\tau\mathcal{P}_0 + 0.7Pt_i(1 - e^{-\frac{t_i}{\tau}}); \\ E_{ij}(t_i, t_j) &= 0.3Pt_j\left(1 - e^{-\frac{t_j}{\tau}}\right) + 0.7Pt_i\left(1 - e^{-\frac{t_i}{\tau}}\right); \\ E(t) &= \sum_{k=1}^n E_k c_k = E_0 c_0 + E_i(t_i) c_i \\ &+ E_{i,j}(t_{i,j}) c_{i,j}, \quad \sum_{k=1}^n c_k = 1, \end{aligned}$$

where  $c_k$  is the coefficient of the contribution of each generating unit operating in the DG-based power system.

## VI. CONCLUSION

The development of DG-based power systems puts forward reliability requirements, which differ from conventional ones. Although the reliability of individual components of electric power systems and power supply systems is growing, the pace of this growth is behind the rate of increase in system complexity.

The research has established that the DG-based power systems are characterized by a variable structure, which can be changed randomly, and by overlapping service areas of generating units, which can have various operating conditions. The nature of the functions and many behaviors of the DG-based power system require its high reliability, but numerous components, their connections, and operating conditions can negatively affect the efficiency of the system as a whole. The absence of practical methods for assessing the efficiency and reliability of such systems hinders the improvement in their competitiveness. Therefore, this area of research is essential.

The efficiency of power systems with distributed generation should be estimated by a specialist who is familiar with the entire system, knows the requirements for it, its structure, operating conditions, methods, and tools for control and protection.

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