A Methodology for Performance and Reliability Analysis of Prosumers' Local Heat Sources in District Heating System

V.A. Stennikov, I.V. Postnikov*, and A.V. Penkovskii

Melentiev Energy Systems Institute of Siberian Branch of Russian Academy of Sciences, Irkutsk, Russia

Abstract **— The paper focuses on two problems addressed in the studies on the prosumer in district heating systems. The first problem is associated with load distribution in the district heating systems with prosumers. The research proposes a bi-level model for solving this problem to determine an optimal balance between the load of district heat sources and prosumer-owned heat sources. The second problem concerns the reliability of heating for consumers to be provided through the optimal distribution of reliability parameters among components of the system, given the capabilities of the prosumer to supply part of load by their heat sources. The methods and models are proposed to solve this problem. They are based on the theory of random processes, theory of hydraulic circuits, and basic laws of cogeneration. The case study results obtained using the developed methodology demonstrate a potential economic benefit and reliability effect of involving the prosumer in heat supply. The conclusions and directions for further research are formulated.**

Index Terms**: District heating system, prosumer, mathematical modeling, optimal operating, bi-level programming, reliability ensuring, nodal reliability indices, markov random process.**

I. INTRODUCTION

The intensive development of energy and information technologies affected greatly the theory of energy system design. The core principles of this theory are the integration of different generators, transition to intelligent systems (i.e., the system's capability to generate and implement solutions based on the forecast and analysis in combination with self-learning), and an increasing involvement of the consumer in energy supply. The latter principle is implemented within the prosumer (producer-consumer) concept. The prosumer functions are regulation and optimization of their demand to enhance the efficiency and reliability of both the prosumer and the entire energy system. Prosumers have their sources and energy storage devices that enable them to vary the amount of power received from the system and its properties (reliability, losses, quality, and others) based on the balance between their needs and capabilities. Researchers in many countries conduct theoretical and practical studies on the prosumer. The investigations to be emphasized are presented in [1–7]. They focus on various aspects of prosumer operation and control within power supply systems. The papers [8] and [9] deal with some issues of prosumer operation in the district heating systems (DHS). Some optimal management options for DHS with prosumers with focus on reliability are considered in [10–12]. Following the analysis of publications on the prosumer topic, which is part of a more general theory of intelligent integrated energy systems (Smart Grid), we can conclude that almost all these investigations concern electric power systems. At the same time, these technologies are also relevant for DHS, which are the largest fuel consumers, especially in Russia, where the fossil fuel consumption for heating needs exceeds 45% of the total values.

One of the main objectives of involving the prosumer

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* Corresponding author.
E-mail: postnikov@isem.irk.ru

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(hereinafter, heat prosumer) in the DHS is to enhance the efficiency and cost-effectiveness of the system by managing the optimal distribution of sources supplying heat loads. The methodological problems to be solved to achieve this objective are various and traditional for energy systems and DHS, in particular. The involvement of prosumers in the DHS, however, brings about new aspects in these problems and calls for new methodological approaches to solve them. The development of these methods is a subject of the presented paper.

II. A BI-LEVEL MODEL FOR MANAGEMENT OF DHS WITH PROSUMERS

The management of DHS with prosumers implies distribution of heat load among system district heat sources (HSs) and local HS that belong to prosumers according to some criteria providing the required (anticipated) parameters of system operation. The inclusion of prosumers with their HSs in the DHS changes the organizational model of the system operation. Thus, the need arises to consider the management at two levels – district heat sources (system level) and the prosumer self-generation (prosumer level).

Mathematically, the problem of managing the DHS with prosumers is solved by the *bi-level programming* methods [13–16]. Such methods are used in different fields of science and technology to solve multi-criteria problems, especially in the case of conflicting interests of the studied subjects. For example, the focus of [16] is on a bi-criterion framework designed to minimize cost and risk together under constraints with the view to determining the optimal error identification and patching time for solving the problem for the software. The management structure of DHS with the prosumer is based on a hierarchical bi-level approach and is as follows. At the first level, we solve the problem of cost minimization for the prosumer. The objective function represents a sum of heat purchase costs and heat production costs for the prosumer-owned HSs. The second level of management corresponds to the system for which the problem of profit maximization is posed. The profit is defined as revenue from selling the heat produced by district heat source less its production costs.

The DHS and prosumer interact as follows: 1) the prosumer submits a bid for the amount of heat they need; 2) the system submits a price offer obtained according to its profit maximization; 3) the prosumer chooses optimal load distribution for load supply from the system (from the district HS) and from prosumer's HS according to the cost minimization to be achieved and makes a bid for the load again. In this cycle, an equilibrium that satisfies both participants of the interaction is determined. The bi-level optimization model of managing the DHS with the prosumer is formulated as follows:

1) the objective functions are:

$$F^{(1)}_{\text{obj}} = \sum_{j\in J}\sum_{\tau\in T}(c^{h}_{j\tau}q^{\text{sys}}_{j\tau} + \alpha_j q^{2}_{j\tau} + \beta_j q_{j\tau} + \gamma_j) \to \min; \quad (1)$$

$$F^{(2)}_{\text{obj}} = \sum_{j\in J}\sum_{\tau\in T}c^{h}_{j\tau}q^{\text{sys}}_{j\tau} - \sum_{i\in I}\sum_{\tau\in T}(\alpha_i q^{2}_{i\tau} + \beta_i q_{i\tau} + \gamma_i) \to \max; \quad (2)$$

2) the load curve of the consumer is:

$$q_{\text{o}j\tau} = q_{\text{o}j}[1-(1-\omega_j)(\tau/\tau_{\text{o}})^{\sigma_j}],\ j\in J,\ \tau\in T; \quad (3)$$

3) the flow distribution model is:

$$\mathbf{A}\mathbf{x}_\tau = \mathbf{g}_\tau,\ \tau\in T; \quad (4)$$

$$\overline{\mathbf{A}}^{\mathrm{T}}\mathbf{p}_\tau = \mathbf{h}_\tau - \mathbf{H}_\tau,\ \tau\in T; \quad (5)$$

$$\mathbf{S}\mathbf{X}_\tau\mathbf{x}_\tau = \mathbf{h}_\tau,\ \tau\in T; \quad (6)$$

4) energy flow balances are:

$$\sum_{i\in I} q_{i\tau} - \sum_{j\in J} q_{\text{o}j\tau} = 0,\ \tau\in T; \quad (7)$$

$$q_{\text{o}j\tau} = q^{\text{sys}}_{j\tau} + q_{j\tau},\ j\in J,\ \tau\in T; \quad (8)$$

5) constraints on variables and parameters are:

$$q_{\text{o}j\tau} > 0,\ q^{\text{sys}}_{j\tau} \ge 0,\ c^{h}_{j\tau} > 0,\ j\in J,\ \tau\in T; \quad (9)$$

$$0 \le q_{j\tau} \le q_{j\max},\ j\in J,\ \tau\in T; \quad (10)$$

$$q_{i\min} \le q_{i\tau} \le q_{i\max},\ i\in I,\ \tau\in T; \quad (11)$$

$$p_{j\min} \le p_{j\tau} \le p_{j\max},\ j\in J,\ \tau\in T. \quad (12)$$

Here: j is a heat consumer; J is a set of heat consumers (including prosumers); i is a district heat source; I is a set of district heat sources; τ is time instant corresponding to the number of hours with a specified load of consumers, h; T is a set of time instants corresponding to the number of hours with a specified load; $F^{(1)}_{\text{obj}}$ is objective function of the first level (consumer), EUR; $F^{(2)}_{\text{obj}}$ is objective function of the second level (system), EUR; $c^{h}_{j\tau}$ is heat price for consumer j at time instant τ, EUR/GJ; $q^{\text{sys}}_{j\tau}$ is part of heat load of consumer j, which is supplied from the system (by district HS) at time instant τ, GJ/h (only for prosumer); $q_{j\tau}$ is part of heat load of consumer j, which is supplied by their heat sources (only for prosumer) at time instant τ, GJ/h; α_j

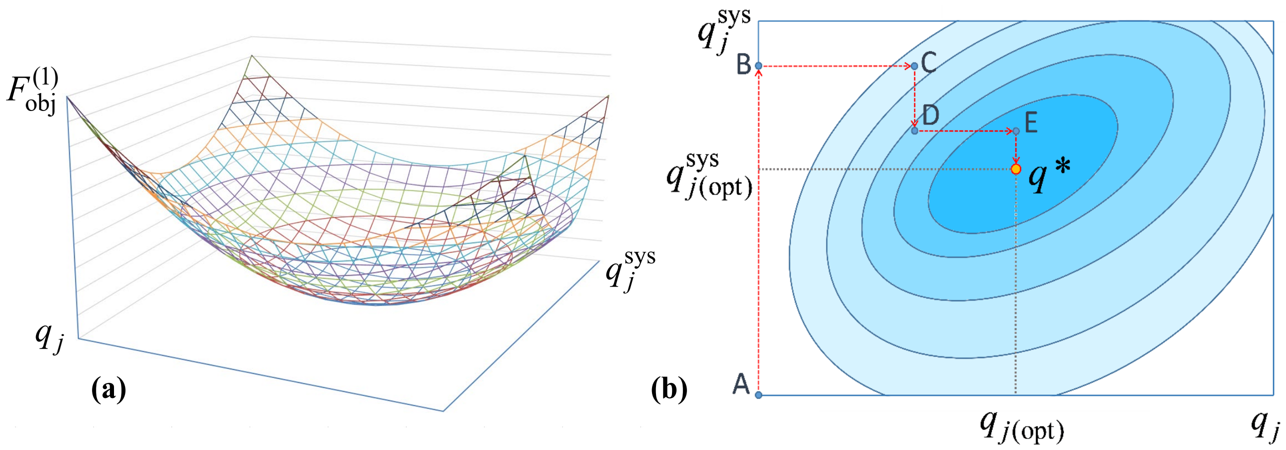

Fig. 1. Graphical illustration of a computational process of search for an optimal solution to the problem of managing DHS with the prosumer: (a) the objective function form is in a three-dimensional system of coordinates; (b) a projection of the objective function plot to the plane of coordinates of heat loads of sources and the solution search procedure.

and α_i, β_j and β_i, γ_j and γ_i are coefficients of cost function for consumer j and the i-th district HS respectively; $q_{i\tau}$ is performance of the i-th district heat source at time instant τ, GJ/h; q_{oj}, $q_{oj\tau}$ are design (maximum) heat load and the heat load of consumer j, which corresponds to time instant τ, GJ/h; ω_j, σ_j are heat load irregularity factors for consumer j; $\mathbf{A}$ is incidence matrix of linearly independent nodes of the heat network (HN); $\overline{\mathbf{A}}^{\mathrm{T}}$ is complete transpose of node-branch incidence matrix for HN; $\mathbf{x}_\tau$ is vector of heat carrier flow rates in the HN branches at time instant τ, t/h; $\mathbf{g}_\tau$ is vector of flow rates at network nodes at time instant τ, t/h; $\mathbf{p}_\tau$ is vector of HN nodal pressure at time instant τ, Pa; $\mathbf{h}_\tau$ is vector of head losses in the network branches at time instant τ, Pa; $\mathbf{H}_\tau$ is vector of operating heads at the sources at time instant τ, Pa; $\mathbf{S}$, $\mathbf{X}_\tau$ are diagonal matrices of coefficients of hydraulic resistances of branches, m/(h^{2}t^2), and absolute values of flow rates in them, t/h, at time instant τ respectively; $q_{i\min}$, $q_{i\max}$ are minimum and maximum values of the i-th district HS capacity, GJ/h; $q_{j\max}$ is maximum value of the HS capacity for consumer j (for prosumer), GJ/h; $p_{j\min}$, $p_{j\max}$ are minimum and maximum values of the heat carrier pressure at nodes of consumer j, Pa. Expression (3) is used to specify a consumer load at each instant of the considered period (a heating season) based on the Rossander equation from [17]. Expressions (4) – (6) represent a model of flow distribution in heat network written in a matrix-node form, which is traditional for the *Theory of hydraulic circuits* (THC) [18]. Parts of expressions in (1) and (2) that define heat production costs at HS are presented as quadratic dependences derived by approximating the real-life data.

With the methods presented in [19–21], the considered problem (1) – (12) can be transformed as follows: Find:

$$F_{\text{obj}\tau}^{(1)} = c_{i\tau}^h q_{i\tau} + \sum_{j \in J_i} (\alpha_j q_{j\tau}^2 + \beta_j q_{j\tau} + \gamma_j) \to \min; \qquad (13)$$

subject to:

$$q_{i\tau} = (c_{i\tau}^h - \beta_i)/2\alpha_i, \, i \in I, \, \tau \in T; \qquad (14)$$

under conditions and constraints (3)–(12).

Here: $F_{\text{obj}\tau}^{(1)}$ is an objective function of the first level determined at time instant τ, EUR/h; $c_{i\tau}^h$ is the price of heat generated at the i-th district HS at time instant τ, EUR/GJ. Thus, after the transformations, the management problem of DHS with the prosumer (13) – (14) with (3) – (12) represents, unlike the initial form (1) – (12), a traditional mathematical programming problem. Figure 1 demonstrates a graphical illustration of the computational process of search for an optimal solution to the management problem of DHS with the prosumer.

The spatial view of the objective function is presented in Fig. 1a and its projection on the system of coordinates of heat loads of the sources – in Fig. 1b. Elliptic lines in this diagram are isocost lines, i.e., the lines of equal costs. The dotted line A-B-C-D-E-$q*$ corresponds to a step-by-step procedure of search for a solution. The formulated problem is solved by the coordinate descent method with a simple iteration within the cycle to reduce the multidimensional optimization problem to a one-dimensional one with a step-by-step procedure for improvement in solutions according to the heat production output by all the sources.

III. METHODS AND MODELS TO ENSURE THE RELIABILITY OF DHS WITH PROSUMERS

3.1. Statement of the problem

The development of methods for reliability analysis and optimization of the DHS with the prosumer generates two main lines of investigations. The first line is related to analysis and optimization of heat supply reliability, considering the prosumer functions in the system, provided the quantity, nodes and capacity of the prosumer sources are specified. The second line represents the problem aimed at determining the number of prosumers, their connection points, and capacities.

This paper focuses on the first reliability problem, which implies the determination of the values of reliability parameters (failure and restoration rates) for the DHS components, which ensure the required reliability level of heat supply to consumers, including prosumers (given their additional redundancy), at a minimum cost of reaching these values within their feasible range. Two main *reliability indices* are assumed to assess the level of heat supply reliability from [22]. These are *failure free operation probability* R_j (FOP) and *availability factor* K_j (AF).

The indices are determined for each consumer $j \in J$, where J is a set of consumers in the system (including prosumers). The standard values of these indices are denoted by R_{oj} and K_{oj}.

The stated problem is solved for these reliability indices according to a technique consisting of 4 stages:

1) determine relationships between the average reliability parameters of DHS components;
2) model the DHS post-emergency conditions;
3) model the prosumer functions;
4) formalize the problem of determining optimal reliability parameters of the DHS components.

3.2. The relationship between the average reliability parameters of DHS components

The average reliability parameter of DHS components is taken to mean their failure or restoration rate that preliminarily has the same value for these components, which provides the required level of reliability indices. These parameters are determined based on expressions for calculation of nodal reliability indices [22], Rossander equation that determines annual heat load curves of consumers from [17], and some basic laws of district heating and thermal physical processes that occur in the system [23]. Based on these expressions, we determine the following relationship between the average reliability parameters of DHS components, given the fulfillment of FOP and AF conditions:

$$\bar{\lambda}_j = \frac{1}{\tau_o}\left(\ln\frac{1}{R_{oj}}\right) \times \left(1 - N_s(1-K_{oj}) \Big/ \sum_{s\in E} L_s^{1/\sigma_j}\right)^{-1} \times \left(\sum_{s\in E} M_s^{1/\sigma_j}\right)^{-1}; \tag{15}$$

where

$$L_s = \frac{1}{1-\omega_j}\left[1 - \frac{1}{q_{oj}}\left(q_{sj} + \varphi_j\left(t_{sj} - \frac{C_1 - C_2\exp B_j}{C_3(1-\exp B_j)}\right)\right)\right]; \tag{16}$$

$$M_s = \frac{1}{1-\omega_j}\left(1 - \bar{q}_{sj} + \frac{\varphi_j t_{sj}}{q_{oj}} - \varphi_j\frac{C_1 - C_2\exp B_j}{C_3 q_{oj}(1-\exp B_j)}\right); \tag{17}$$

$$C_1 = t_{oj}(1-\bar{q}_{sj}),\ C_2 = t_{j\min} - t_{oj}\bar{q}_{sj},\ C_3 = 1 - \bar{q}_{sj}; \tag{18}$$

$$\bar{q}_{sj} = q_{oj} / q_{sj},\ s \in E,\ j \in J; \tag{19}$$

$$B_j = 1/(\varepsilon_j \bar{\mu}_j),\ j \in J. \tag{20}$$

Here: $\bar{\lambda}_j$ and $\bar{\mu}_j$ are average failure and restoration rates for consumer j respectively, 1/h; τ_o is time instant corresponding to a total number of hours of the considered period (heating season), h; N_s is the quantity of system states; s is number of system state; E is a set of system states; ω_j, σ_j are irregularity factors of heat load curve of consumer j [17]; q_{oj} is design (maximum) heat load of consumer j, GJ/h; q_{sj} is level of heat supply to consumer j in system state s, GJ/h; φ_j is coefficient of specific heat losses for consumer j, GJ/(h°C); t_{sj} is current internal temperature for consumer j in state s, °C; $\bar{q}_{sj}$ is relative heat supply to consumer j in system state s, GJ/h; t_{oj} is design temperature of internal air for consumer j, °C; $t_{j\min}$ is minimum admissible temperature of internal air for consumer j, °C; ε_j is coefficient of thermal energy storage for consumer j, h; L_s, M_s, C_1, C_2, C_3, B_j are assumed abbreviations of expressions.

3.3. Modeling the prosumer functions

The prosumer functions in the emergency conditions, which limit or totally terminate heat supply, are to eliminate heat undersupply by their source and increase time redundancy. These properties of the prosumer are considered in the calculation of indices $\bar{q}_{sj}$ and B_j. The following expressions are added to equations (19) and (20):

$$\bar{q}_{sj} = q_{oj} / (q_{sj}^{\text{sys}} + q'_{sj}),\ s \in E,\ j \in J; \tag{21}$$

$$B_j = 1/[(\varepsilon_j + \Delta\varepsilon_j)\bar{\mu}_j],\ j \in J. \tag{22}$$

Here: q_{sj}^{sys} is part of heat load of consumer *j*, supplied from the system (by district HS) in system state *s*, GJ/h; q'_{sj} is capacity of HS for consumer *j* (prosumer) in system state *s*, GJ/h; $\Delta\varepsilon_j$ is additional passive time reserve of prosumer (caused by the use of their heat generators and/or heat storage devices), h. It is worth noting that time reserve (both active and passive) is one of the most effective methods to improve the reliability of functioning of various technical systems. Therefore, for example, in the study [24], based on the theory of semi-markov processes with a common phase space of states, a semi-markov model of a multicomponent system with a group instantly replenished time reserve is constructed. The index q'_{sj} can be fixed and correspond to a rated (required) value of capacity of the prosumer HS. This index can vary and factor in the component failures decreasing the heat source performance. Here, corresponding failures should be added to a set of system states. In this case, the solution should be based on an analysis of the initial characteristics of equipment reliability. If the failure flow values for the equipment of the prosumer heat source are much lower than those for the DHS equipment, then the failures at the prosumer heat source can be neglected and value q'_{sj} can be assumed to be fixed according to their required performance under specified emergency heat supply to consumers.

3.4. Modeling the post-emergency conditions

Post-emergency hydraulic conditions are determined by flow distribution in the heat network after the disconnection of a failed component. The hydraulic conditions are calculated by the THC methods [18]. The nodal form of the model of flow distribution (hydraulic conditions) in the heat network is represented by a matrix form [18]:

$$\mathbf{A}\mathbf{x}_s = \mathbf{g}_s,\ s \in E; \tag{23}$$

$$\overline{\mathbf{A}}_s^{\mathrm{T}}\mathbf{p}_s = \mathbf{h}_s - \mathbf{H}_s,\ s \in E; \tag{24}$$

$$\mathbf{S}\mathbf{X}_s\mathbf{x}_s = \mathbf{h}_s,\ s \in E. \tag{25}$$

Here: $\mathbf{A}_s$ is incidence matrix of linearly independent nodes in the network under emergency system state *s* (considering failure of some component); $\overline{\mathbf{A}}_s^{\mathrm{T}}$ is full transposed node-branch incidence matrix; $\mathbf{x}_s$ is vector of heat carrier flow rates in the network sections (branches) under emergency system state *s*, t/h; $\mathbf{g}_s$ is vector of flow rates at network nodes under emergency system state *s*, t/h; $\mathbf{p}_s$ is vector of nodal pressures of HN under emergency state *s*, Pa; $\mathbf{h}_s$ is vector of heat losses in the sections under the emergency system state *s*, Pa; $\mathbf{H}_s$ is vector of operating heads at sources in the emergency system state *s*, Pa; $\mathbf{S}$, $\mathbf{X}_s$ are diagonal matrices of coefficients of hydraulic resistance of sections, m/(h^{2}t^2), which are built from the values of hydraulic resistances of sections and absolute values of flow rates in them, t/h. Modeling of an emergency situation in some state *s* of the system is performed by excluding a component whose failure corresponds to this state from the calculation model of DHS.

3.5. Formalization of a problem of determination of optimal reliability parameters of DHS components

The values of average reliability parameters determined by expressions (15)–(20) are distributed among the system components according to the following equations of conservation of the sum of system state probabilities:

$$\overline{\lambda}_j \sum_{s\in E} p_s = \sum_{n\in N} \sum_{s\in E(n)} \lambda_n p_s,\ j \in J; \tag{26}$$

$$\overline{\mu}_j \sum_{s\in E} p_s = \sum_{n\in N} \sum_{s\in E(n)} \mu_n p_s,\ j \in J. \tag{27}$$

Here: p_s is probability of system state *s*; *n* is number of system component; *N* is a set of system components; *E*(*n*) is a subset of system states to which the system can transition because of failure or restoration of component *n*; λ_n, μ_n are failure or restoration rates of component *n*, 1/h. The probabilities of system states are determined by solving the *markov random process* equations describing a sequence and structure of events that characterize the DHS operation. The use of the markov processes for reliability analysis of a considered object is justified by a large number of applications of this mathematical tool for studying the reliability of various energy systems. The methodological issues related to the application of markov random processes for reliability analysis of DHS subsystems (HS and HN) are discussed in [22, 25–29]. The studies discussed in [30–33] apply the markov models for the comprehensive analysis and optimization of the DHS reliability, considering all stages of the technological process of thermal energy production and distribution.

Stationary markov model of DHS operation can be represented by the following system of linear equations:

$$p_s\left(\sum_{n\in N(s)}\lambda_n + \sum_{n\in N(s)}\mu_n\right) = \sum_{z\in E(s)}\left(\sum_{n\in N(z)} p_z\lambda_n + \sum_{n\in N(z)} p_z\mu_n\right),\ s \in E. \tag{28}$$

Here: p_z is probability of system state z (division of state into s and z is necessary to write the system of equations of random process); $N(s)$ is a subset of system components whose failure or restoration corresponds to a direct transition of the system from state s to some other state z; $N(z)$ is a subset of system components whose failure or restoration corresponds to a direct transition of the system from state z to some other state s; $E(s)$ is a subset of the system states from which the system can transition to s.

According to the problem stated in point 3.1, the objective function of DHS reliability optimization is expressed as a sum of costs required to ensure the values of reliability parameters of the DHS components:

$$F_{\text{obj}} = \sum_{n\in N} f_{n\lambda}(\lambda_n) + \sum_{n\in N} f_{n\mu}(\mu_n). \qquad (29)$$

Here: $f_{n\lambda}(\lambda_n)$, $f_{n\mu}(\mu_n)$ are cost functions of ensuring reliability parameters of components, i.e., their failure and restoration rates respectively, EUR. The type of functions (29) and their quantitative parameters are determined by the methods of approximation on the basis of an analysis of the actual data on the cost of equipment with different reliability characteristics, costs of installing the backup components; costs of establishing and maintaining the emergency and restoration services, and other measures capable to improve the reliability.

Technically possible values of reliability parameters of the components are specified by the following constraints:

$$\lambda_n^{\min} \le \lambda_n \le \lambda_n^{\max},\ n \in N; \qquad (30)$$

$$\mu_n^{\min} \le \mu_n \le \mu_n^{\max},\ n \in N. \qquad (31)$$

Thus, the optimization problem of DHS component reliability, given prosumers, lies in the following:

Minimize function (29) under the following conditions and constraints:

1) conditions (15) – (20), which determine the relations between average reliability parameters of DHS components in terms of the prosumer functions (21), (22);
2) conditions (23) – (25) according to which the levels of heat supply under different system states are calculated;
3) conditions (26) and (27), which determine the principles of allocating the average reliability parameters among the system components;
4) condition (28), which determines the probabilities of states by solving equations of the markov random process;
5) constraints (30) and (31) on the reliability parameters of components.

The nonlinearity of the considered optimization problem is connected firstly with function (29), which is usually a power function. Such problems are solved by the iteration methods using the GAMS and Maple software as solver.

IV. COMPUTATIONAL EXPERIMENT

The computational experiment is conducted for the DHS diagram presented in Fig. 2a. The diagram consists of two district heat sources (HS1 and HS2), seven consumers (nodes 1–7), and a looped HN consisting of 18 sections (branches). The overall heat load of DHS is 2 341 GJ/h, and the total capacity of district sources is 2 717 GJ/h. One of the consumers is a prosumer (P7) with a load of 400 GJ/h and HS with a capacity of 200 GJ/h. HS2 supplies heat to P7 according to the hydraulic calculation of the system. Therefore, the problem of optimal management of HS load distribution is solved at the level of interaction between these entities. Figure 2b demonstrates a scheme of their interaction according to the above-described approach and model. P7 is assumed to have its HS running on fossil fuel, for which a power cost function with corresponding approximation coefficients is given. Diagrams of HS1 and HS2 are the same and are shown in Fig. 2c. These diagrams consist of the main aggregate components: boiler 19, turbine 20, network heaters 21 and 23, and network pumps 22 and 24. The indicated component numbers correspond to HS1, the assigned numbers for HS2 are from 25 to 30. The optimal loads for HS2 and P7 in the considered DHS diagram during the entire heating period of 6 000 h with a step of 1000 h are illustrated in Fig. 3a. This diagram shows the heat load curves of P7 with highlighted amounts of heat generated by HS2 and HS of P7. According to Fig. 3a, HS of P7 operates during the time of consumer peak loads and supplies heat in the amount of about 180 GJ/h. The total heat consumption of P7 during the heating period is 1 026.5 thousand GJ, including 890.7 thousand GJ (86.8%) from the district HS2, and 135.9 thousand GJ (13.2%) from self-generation. The economic effect of the P7 source operation can be seen in Fig. 3b. Its value corresponds to the area of a solid figure of orange color and equals EUR 277 thousand or 7.6% of the total cost for the heating period.

A reliability study of the considered system is based on a joined diagram of the heat network (a set of sections) and district sources according to the comprehensive DHS reliability analysis approach presented in [30–32]. The random process of the DHS functioning is modelled under

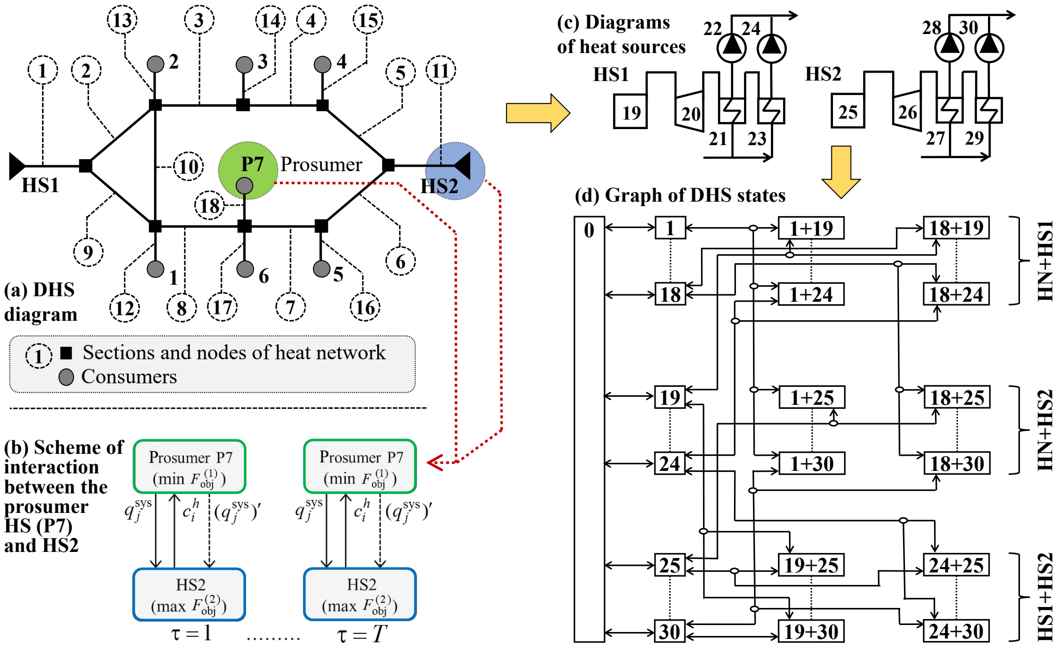

Fig. 2. Illustration of the computational experiment: (a) general diagram of tested DHS; (b) scheme of bi-level interaction between the prosumer HS (P7) and the district HS2; (c) principal diagrams of district HS1 and HS2; (d) graph of DHS states and transitions between them.

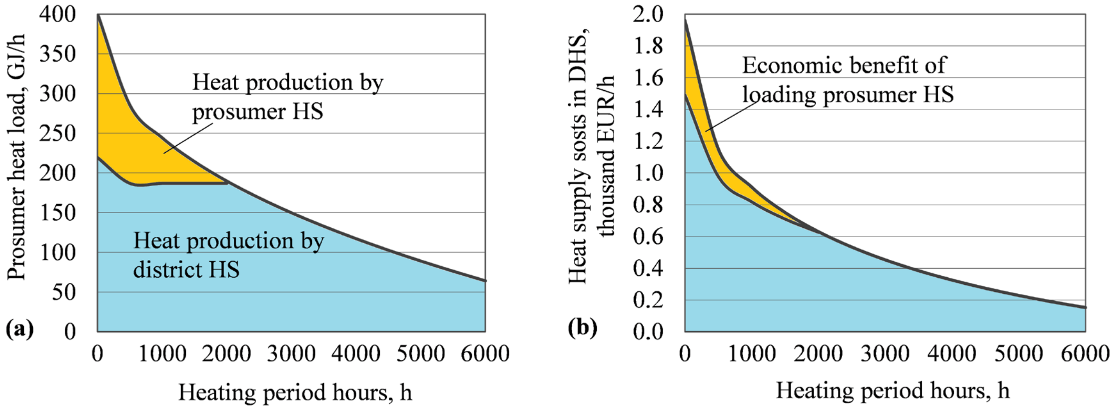

Fig. 3. Results of the optimal management of considered DHS with prosumer: (a) economically optimal load of prosumer HS (P7) and district HS2 during the heating period: (b) a graph of heat production costs in DHS with benefit of prosumer P7.

the condition of simple flow of events (Poisson flow), and the formation of a set of states is limited by consideration of the joint failure of no more than two components from different DHS subsystems (HN, HS1, and HS2). The oriented graph corresponding to this structure of states and transitions between them is shown in Fig. 2d. The state numbers on the graph correspond to the failed components in accordance with the diagrams in Fig. 2a and Fig. 2c. The probabilities of these states are determined by solving a system of 283 equations of the markov process of form (28). Optimization of the reliability parameters of system components is carried out when the following normative (required) values of the nodal reliability indices are met [22]: AF = 0.97 and FOP = 0.905. The ranges for possible values of the optimized reliability parameters of system components are set as follows: failure rate is 0.0002–0.0025 1/h; restoration rate is 0.007–0.09 1/h. Power-law functions of costs for ensuring reliability parameters, which form the objective function (29), were obtained on the basis of approximation of reference data on the structure and unit cost of reserved components, and the emergency services [22].

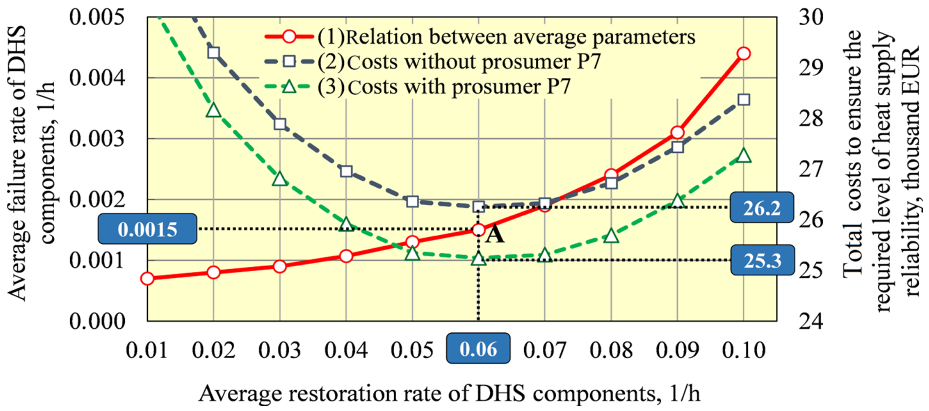

Fig. 4. Results of ensuring reliability of DHS with prosumer: optimal relation between average (integrated) reliability parameters of system components (failure and restoration rates) to fulfill the requirements for the reliability indices at minimal reliability cost.

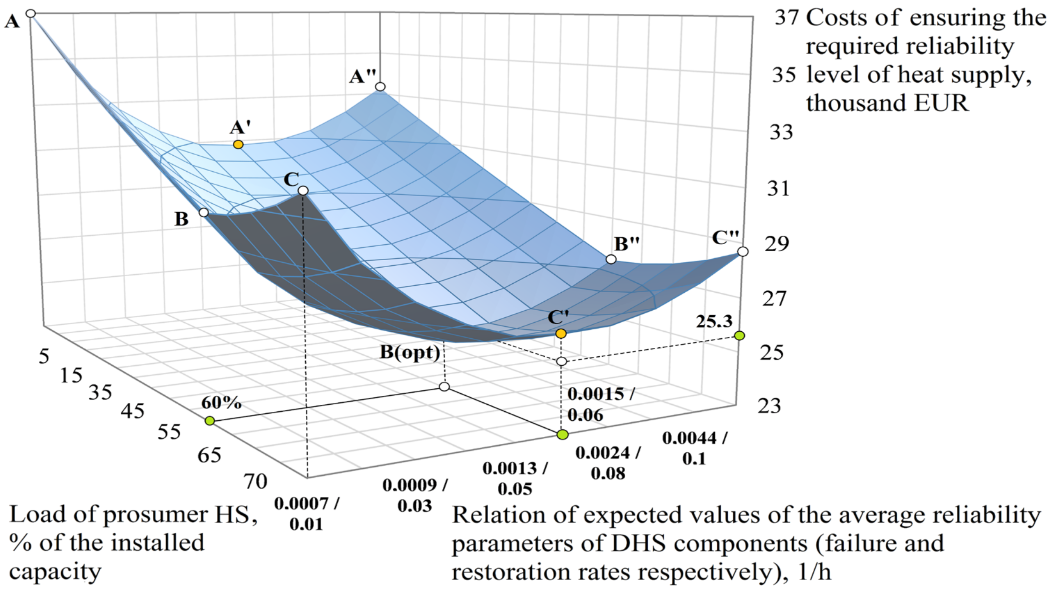

Fig. 5. Results of a comprehensive reliability study for prosumer functioning in DHS: relation between reliability parameters of system components (failure and restoration rates) and costs of ensuring the required reliability level of heat supply depending on the load of prosumer P7 source.

Figure 4 shows the result of search for the optimal relation of the average (integrated) reliability parameters of system components (graph 1), which correspond to the minimum costs of ensuring the reliability of the system when performing the required reliability indices (AF and FOP). Graphs 2 and 3 in Fig. 4 present the change in the costs of ensuring the required level of reliability with and without considering the functioning of P7 source respectively. The obtained solution at point A corresponds to the following values of average (integrated) reliability parameters: failure rate – 0.0012 1/h, restoration rate – 0.048 1/h. These values of integrated parameters are distributed over the system components according to equations (26) and (27) in the following ranges of values: 0.0004–0.0016 1/h for failure rate and 0.025–0.07 1/h for restoration rate. The cost of ensuring reliability is EUR 26.2 million without reserve of P7 source and EUR 25.3 thousand with reserve of P7 source. The economic effect from the use of reserve by source of P7 in the system is EUR 0.9 thousand or 3.4%.

Figure 5 illustrates a search for an optimal reliability solution for DHS given the use of capacity and time reserve of the prosumer HS (P7). This diagram shows the relation between reliability parameters of DHS components (failure and restoration rates) and costs of ensuring the required reliability level of heat supply depending on the load of P7 source. The obtained solution at point B(opt) corresponds to the costs necessary to ensure reliability in the amount of EUR 25.3 thousand with a determined relation of expected values of reliability parameters (failure rate is 0.015 1/h and restoration rate is 0.06 1/h) and loading for the P7 heat source at the level of 60%.

The results obtained allow outlining some features of the prosumer functioning in the heating system. In particular, we can conclude that the most effective area of distributed HS application corresponds not to the full coverage of consumer loads but to the level of 60–80% (depending on the reliability requirements). This provides a decrease in the consumer load on the system and compensates for the lack of thermal energy under emergency conditions in the system (in case of a failure of the system components). Both of these significantly increase the reliability of heat supply to prosumers themselves and the reliability of the entire system, embracing consumers that do not have heat self-generation and other active reserves. In addition, during the period of maximum heat loads (at minimum outside temperatures), the prosumer HS can operate as a peak source, reducing the load on district heat sources and lowering the likelihood of emergency conditions associated with failure of equipment operating at limiting operating parameters in such periods.

V. CONCLUSION

The significance of the studies on heat prosumer is related to the objective problems in heat supply, including low cost-effectiveness of operating DHS and insufficient quality and reliability of heat supply to consumers. This research states the load management problem of DHS with prosumers. Solving this problem is aimed at the cost-effective distribution of heat sources to supply heat to consumers from both district heat source and prosumer-owned heat source. A mathematical tool of bi-level programming is used to solve this problem. The second research problem is formulated to ensure DHS reliability considering the prosumer functions as a way to provide additional capacity and time reserve owing to the prosumer heat source. This problem is solved using nodal reliability indices, markov random process, some basic laws of cogeneration, and others.

The scientific novelty of the conducted research lies in the following:

1) The problem of managing the joint operation of district heating system (DHS) and distributed prosumer heat sources (HSs) within the DHS was formulated for the first time as a bi-level optimization of the heat load of these sources;

2) A principle of the most rational solution (equilibrium) for the distribution of heat loads of district HSs and distributed prosumer HS in DHS was proposed based on a three-stage cycle of strategies of the heat market actor;

3) A bi-level mathematical model was developed to optimally manage joint operation of district HS and distributed prosumer HS in DHS (loading of district and distributed sources) based on technical and economic criteria;

4) The bi-level mathematical model was modified to a single-extreme optimization problem, which simplifies the calculations;

5) The problem of ensuring the DHS reliability in terms of the redundancy functions of prosumer (mainly by its sources) is formulated, which involves searching for an optimal relation of the active power reserve of the prosumer's sources and the functional reserve of the system (component reliability);

6) A technique is proposed for ensuring reliability, based on various methods and models: models of markov random processes, methods of the theory of hydraulic circuits, analytical expressions describing the change in heat loads, the processes of accumulating thermal energy (thermal inertia), etc.;

7) The mathematical model is obtained to determine the average (integrated) reliability parameters of system components, ensuring the requirements for the reliability indices of heat supply to consumers, given the factors specified above in point 6.

The main advantage of the model designed to optimally manage the jointly functioning district HSs and distributed prosumer HS in DHS (point 3), in comparison with the existing developments, is the comprehensive consideration of various technical and economic aspects of the studied systems operation, including thermo-hydraulic conditions in the network (flow distribution), operating costs for the production and distribution of thermal energy in the system, change in heat loads during the design period, and some others.

The main advantage of the proposed methods for ensuring the reliability (point 6) lies in the integration of measures aimed at reducing the failure rates and enhancing restoration rates of the components in a joint procedure of search for reliability parameters, which makes it possible to most rationally distribute the overall potential enabling the increase in reliability of DHS components. The practical applicability of the developed methods for optimization of reliability parameters of DHS components is confirmed by the calculations carried out on a test diagram.

The results obtained in the calculation for the test DHS diagram show the operability of the developed mathematical model and the possibility of gaining the economic and reliability effect owing to the use of the prosumer heat source. Further research in this area will provide a more informed assessment of the effectiveness of the prosumer involvement in district heating related to economical and reliability.

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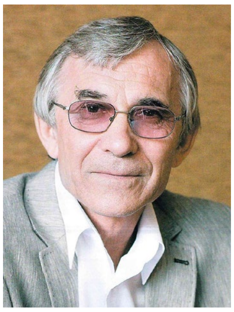

Valery A. Stennikov, Academician of the Russian Academy of Sciences (RAS), Dr. Eng., Professor, Director of Melentiev Energy Systems Institute of Siberian Branch of the Russian Academy of Sciences (ESI SB RAS), Irkutsk, Russia. Valery Stennikov is an author and co-author of more than 500 scientific publications. The main research areas are systems research in the energy industry, theory of hydraulic circuits, mathematical modeling, optimization methods, heat supply systems, district heating systems, cogeneration, reliability, energy efficiency, energy saving, and intelligent energy systems.

Ivan V. Postnikov, Ph.D., Senior Researcher in the Laboratory of Heat Supply Systems of ESI SB RAS, Irkutsk, Russia. He received the Ph.D. degree from ESI SB RAS in 2013. Ivan Postnikov is an author and co-author of more than 130 scientific publications. The main research areas are energy systems, district heating systems, mathematical modeling, reliability, random processes, optimization, energy efficiency, intelligent energy systems, prosumer, and renewable energy.

Andrey V. Penkovskii, Ph.D., Senior Researcher in the Laboratory of Heat Supply Systems of ESI SB RAS, Irkutsk, Russia. He received the Ph.D. degree from ESI SB RAS in 2017. Andrey Penkovskii is an author and co-author of more than 120 scientific publications. The main research areas are energy systems, district heating systems, mathematical modeling, energy market, optimization methods application, energy efficiency, prosumer, and game theory.