A Novel Technique for Technical and Non-Technical Power Loss Control and Monitoring in Power Distribution Systems as Based on the Data from the Automated System for Electricity Revenue Metering

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***Abstract* — Uncontrolled power losses caused by electricity thefts in power distribution systems (PDSs) have a detrimental impact on technical and economic performance of these systems. Modern automated systems for electricity revenue metering (ASERMs), which are nowadays widely used for automation and computerization of processes in PDSs, lack digital technologies for separate estimation and monitoring of technical and non-technical power losses. The paper proposes a novel technique and an algorithm for its implementation based on the data obtained from electricity meters that are part of the ASERM structure. In order to ensure the solvability of the identification problem, we introduce the concept of a virtual network model representing the desired state of the PDS in the absence of ETs. The model serves as the basis for deriving algebraic equations. Such equations allow identifying technical and non-technical power losses in three-phase networks. Our research findings aim at further improvement of modern ASERMs and enhancement of their performance and reliability.**

***Index Terms*— Power distribution system, power loss, detection and monitoring algorithm.**

I. INTRODUCTION

Automation of information processes in power distribution systems (PDSs) is currently carried out by introducing automated systems for electricity revenue metering (ASERMs) [1], which are part of the Smart Grid infrastructure [2]. As it is known, integrated software and hardware systems of ASERMs belong to the class of meter data management systems, the main function of which is revenue metering of electricity in PDSs. The analysis of the functional structure of existing ASERMs reveals that they fail to separately identify and perform real-time monitoring of technical and non-technical power losses in the network [3] and they do not carry out optimization [4–7], which compromises their efficiency and technical and economic performance of distribution utilities. This is partly due to the fact that proper and instrumental techniques have not

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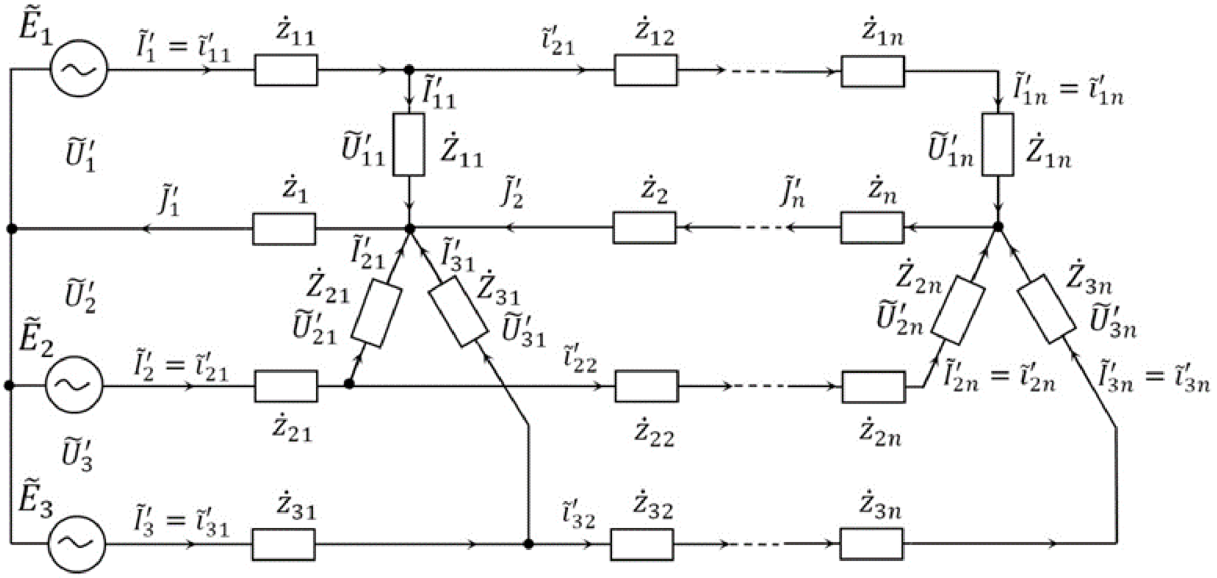


Fig. 1. Notional equivalent circuit of a three-phase network.

been sufficiently developed to solve the above problems to date. By proper techniques we understand those capable of factoring in electricity thefts (ETs) in the system [8–10] and unbalanced load flows in PDSs [11–14]. Another issue to be addressed is that most PDSs are large dynamic systems, which have a complex structure and operate under incomplete information about their states and parameters. The papers [15, 16] present several approaches to power loss detection in 0.4 kV distribution systems. This paper discusses algorithmic problems associated with elaboration of those approaches.

II. PROBLEM STATEMENT

The study centers on a four-wire 0.4 kV PDS, the simplified equivalent circuit of which is shown in Fig. 1. The notation used in the figure are: k, ν are index variables denoting the numbers of phases A, B, C ($k=\overline{1,3}$) and electrical circuits of the network, respectively ($\nu=\overline{1,n}$); $\tilde{E}_k$ is the instantaneous electromotive force of the power source for the k-th phase; $\tilde{I}'_{k\nu}$, $\tilde{U}'_{k\nu}$, $\dot{Z}_{k\nu}$ are instantaneous values of sine wave current, voltage, and impedance of the customer's load with the coordinate (k, ν), respectively; $\tilde{i}'_{k\nu}$, $\dot{z}_{k\nu}$ are instantaneous values of sine wave current and impedance of the ν-th inter-customer section of the k-th phase; $\tilde{J}'_\nu$, $\dot{z}_\nu$ are instantaneous values of current and impedance of the ν-th section of the neutral; $\tilde{I}'_k$, $\tilde{U}'_k$ are instantaneous values of current and voltage at the inputs of the corresponding phases.

The following conditions are assumed to be satisfied:

1. The power distribution system operates under unbalanced conditions.
2. There are electricity thefts (ETs) in the network.
3. The cross sections of the phase and neutral wires are the same, i.e., $\dot{z}_{k\nu}=\dot{z}_\nu$.
4. At a discrete moment of time $t=t_\xi$, the electricity meters of ASERM synchronously measure the effective values of currents I'_k, $I'_{k\nu}$ and voltages U'_k, $U'_{k\nu}$ at the phase inputs and at network loads, respectively. The power factors $c_k=\cos\varphi'_k$ and $c_{k\nu}=\cos\varphi'_{k\nu}$ between them are also measured. The obtained information is entered into the ASERM database and vectors of input phase currents $I'=\left[I'_1,I'_2,I'_3\right]$ are formed on their basis. The above data are used to determine the active P_k, $P_{k\nu}$ and reactive Q_k, $Q_{k\nu}$ powers consumed by phases and customers of the network, respectively, by relying on known equations.

At the time ($t=t_\xi$) of synchronized readings of electricity meters included in the ASERM, the distribution system may be in a default (normal, desired) (C^0) or disturbed (C') state. In the first case (state C^0), there are no uncontrolled power losses in the network. In the disturbed state (C') of the network, at least one of its phases has uncontrolled power losses. In this case, the following balance relations hold for powers:

$$\dot{S}_k(\xi)=\dot{S}^a_k(\xi)+\dot{S}^T_k(\xi)+\Delta\dot{S}_k(\xi),\ k=\overline{1,3},\qquad (1)$$

where $\dot S_k(\xi)$ is complex power consumed by the k-th phase at the time $t = t_\xi$; $\dot S_k^a(\xi)$ is total complex power consumed by customers of the k-th phase; $\dot S_k^T(\xi)$ is technical power losses in the k-th phase; $\Delta\dot S_k$ is uncontrolled power losses in the k-th phase of the network, caused by ETs in the PDS.

At the same time, powers $\dot S_k(\xi)$ and $\dot S_k^a(\xi)$ are known values, as existing ASERMs enable the calculation of their values from the data of active and reactive power obtained from the main electricity meter and customers' meters according to the known equations:

$$\dot S_k(\xi) = P_k(\xi) + jQ_k(\xi), \tag{2}$$

$$\dot S_k^a(\xi) = \sum_{\nu=1}^{n} \dot S_{k\nu},\ k = \overline{1,3}, \tag{3}$$

where $\dot S_{k\nu} = P_{k\nu} + jQ_{k\nu}$; $j = \sqrt{-1}$ is an imaginary number.

It is worth noting that traditional ASERMs do not detect separately technical power losses $\dot S_k^T(\xi)$ and non-technical power losses $\Delta\dot S_k(\xi)$. Existing ASERMs estimate only the sum total of power losses $\dot S_k^\Sigma(\xi)$ based on metering data, i.e.,

$$\dot S_k^\Sigma(\xi) = \dot S_k^T(\xi) + \Delta\dot S_\kappa(\xi) = \dot S_k(\xi) - \dot S_k^a(\xi), k = \overline{1,3}.$$

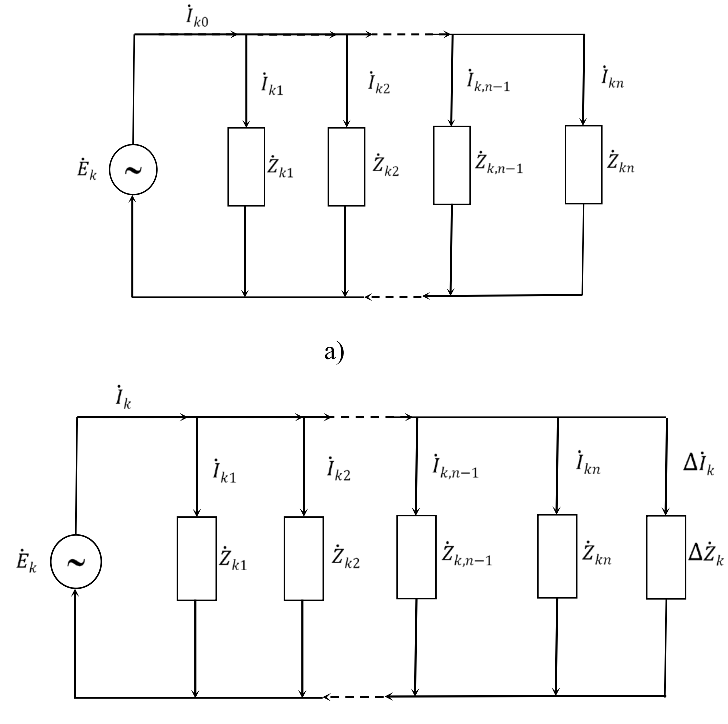
The objective is to detect non-technical $\Delta\dot S_k(\xi)$ and technical $\dot S_k^T(\xi)$ power losses in real time on the basis of metering data obtained from ASERM electricity meters.

Solving the above-stated detection problem includes the following main stages:

1. Build a virtual network (VN) model.
2. Identify the current state of the physical network.
3. Establish the overall procedure for power loss detection in the PDS.
4. Estimate the desired input phase currents of the virtual network.
5. Estimate currents of unauthorized consumers.
6. Design an algorithm for power loss detection in the network.

III. Building a Virtual Network Model

It is worth noting that the investigated detection problem is the one where the input data and conditions for solving it are not fully specified, i.e., the problem should be solved under insufficient information about the state of the physical network. This uncertainty is due to the lack of data



a)

b)

Fig. 2. Virtual network diagrams.

on unauthorized consumers, such as their coordinates and load parameters. In order to solve the detection problem under these conditions, we will introduce the concept of the virtual network (VN) model, which describes the desired state of the physical PDS under certain conditions. To this end, it is reasonable to introduce two types of virtual networks (Fig. 2):

1. Virtual network of the first type (VN$_1$) has no uncontrolled power losses $\Delta\dot{S}_k$ and zero end-to-end impedances $\dot{z}_{k\nu}$ (Fig. 2a):

$$\Delta\dot{S}_\kappa = 0,\ k=\overline{1,3},\quad \dot{z}_\nu = 0,\ \nu=\overline{1,n}. \qquad (4)$$

2. Virtual network of the second type (VN$_2$) is the one with ETs, i.e., the following conditions are met (Fig. 2b):

$$\Delta\dot{S}_k \neq 0,\ k=\overline{1,3}, \qquad (5)$$

and relations (4) hold. Here $\dot{E}_k$ is complex electromotive force of the network power source; $\dot{I}_{k\nu}$, $\dot{Z}_{k\nu}$ are complex current and load impedance of the virtual customer having the coordinate (k, ν); $\dot{I}_{k0}$, $\dot{I}_k$ are complex currents at the inputs of the k-th phase of VN$_1$ and VN$_2$, respectively; $\Delta\dot{I}_k$, $\Delta\dot{Z}_k$ are complex current and load impedance of the unauthorized consumer connected to the corresponding phase of the PDS. Next, without loss of generality of the problem and for the sake of brevity, let us consider the k-th phase of a virtual network to which n customers are connected.

Input phase currents $\dot{I}_{k0}$ of VN$_1$ indicate the default operation of the considered real-life distribution system with no uncontrolled currents $\Delta\dot{I}_k$. Therefore, in what follows, we will call them the desired phase currents. A comparative analysis of the structures of the original PDS (Fig. 1) and virtual networks (Fig. 2) subject to conditions (4) and (5) shows that the following relations hold for the complex currents $\dot{I}_k$, $\dot{I}_{k0}$ and $\Delta\dot{I}_k$:

$$\dot{I}_{k0} = \sum_{\nu=1}^{n} \dot{I}'_{k\nu},\ k=\overline{1,3}, \qquad (6)$$

$$\dot{I}_k = \dot{I}_{k0} + \Delta\dot{I}_k,\ k=\overline{1,3}, \qquad (7)$$

where $\dot{I}_k = \dot{I}'_k$.

The values of currents $\dot{I}_{k\nu}$ and voltages $\dot{U}_{k\nu}$, describing the states of virtual networks, differ from their corresponding values indicating the state of the original physical PDS (Fig. 1), i.e., $\dot{I}_{k\nu} \neq \dot{I}'_{k\nu}$, $\dot{U}_{k\nu} \neq \dot{U}'_{k\nu}$. In this case, the values of load impedances $\dot{Z}_{k\nu}$ of customers of physical and virtual networks have the same values. In what follows, the above-mentioned network variables will be expressed in a complex form as follows [17]:

$$\dot{I}_{k\nu} = I_{k\nu}\, e^{j(\beta_k+\alpha_{k\nu})}, \qquad (8)$$

$$\dot{Z}_{k\nu} = Z_{k\nu}\, e^{j\varphi_{k\nu}},\ k=\overline{1,3},\ \nu=\overline{1,n},$$

where $I_{k\nu}$, $Z_{k\nu}$ are magnitudes of corresponding complex quantities; $\varphi_{k\nu}$ is the argument of impedance $\dot{Z}_{k\nu}$; $\alpha_{k\nu}$ is deviation of phase shifts from their baseline values β_k determined using the equation $\beta_k = \dfrac{2(k-1)\pi}{3}$.

Impedances $\dot{Z}_{k\nu}$ of network customers' loads are known values, which are calculated on the basis of readings (data) of customers' electricity meters by the following equations:

$$Z_{k\nu} = \frac{U'_{k\nu}}{I'_{k\nu}},\ \varphi_{k\nu} = \arccos\left(c_{k\nu}\right). \qquad (9)$$

Corresponding admittances $\dot{Y}_{k\nu}$ of network customers' loads are:

$$\dot{Y}_{k\nu} = \frac{1}{\dot{Z}_{k\nu}} = Y_{k\nu} e^{-j\varphi_{k\nu}},\ k=\overline{1,3},\ \nu=\overline{1,n}, \qquad (10)$$

where $Y_{k\nu} = \dfrac{1}{Z_{k\nu}}$. At the same time, the powers $\dot{S}_{k\nu}$ consumed by the network customers and included in expressions (3) are determined by the equations:

$$\dot{S}_{k\nu} = \left(I'_{k\nu}\right)^2 \dot{Z}_{k\nu},\ k=\overline{1,3},\ \nu=\overline{1,n}. \qquad (11)$$

IV. IDENTIFICATION OF THE CURRENT STATE OF THE PHYSICAL NETWORK

Given the conditions of the stated problem, the current values of phase currents $I'_k = I'_k\left(t_0\right)$ at the input of the real-life physical distribution system (Fig. 1) at the time $t = t_0$ are measured by the main three-phase meter. These data are stored in the ASERM database in the form of the vector $I' = \left[I'_1, I'_2, I'_3\right]$. The complex currents of unauthorized consumers $\Delta\dot{I}_k$ are written in the form:

$$\Delta\dot{I}_k = \Delta I_k\, e^{j(\beta_k+\Delta\alpha_k)},\ k=\overline{1,3}, \qquad (12)$$

where ΔI_k, $\Delta\alpha_k$ are magnitudes and phases of complex currents $\Delta\dot{I}_k$, respectively. Next, we will estimate the effective values of ΔI_k.

Our analysis shows that it is reasonable to use the following conditions to identify the current state of the PDS:

$$\Delta I_k \leq \Delta I,\ k=\overline{1,3}, \qquad (13)$$

where ΔI is the maximum permissible error of current measurement in the ASERM.

Obviously, if at least one of the relations (13) does not hold, there are ETs in the network, whereas their validity means that the PDS operates normally. Thus, relations (13) can be used as a criterion for identification of the current state of the PDS.

V. A General Procedure for Power Loss Detection

Consideration is given to the virtual network diagrams shown in Fig. 2. We use $\dot{S}_k'$ to denote the powers consumed by the loads of the k-th phase of VN$_2$. The analysis shows that the following balance relations exist between powers $\dot{S}_k'$, $\dot{S}_k^a$ and $\Delta\dot{S}_k$:

$$\dot{S}_k^a + \Delta\dot{S}_k = \dot{S}_k', \; k = \overline{1,3}, \tag{14}$$

where the total power $\dot{S}_k^a$ consumed by the loads of the k-th phase is determined by equation (3). Each of these components, when conditions (4) and (5) are satisfied, is determined by the following functional relations:

$$\begin{aligned} \dot{S}_k^a &= I_{k0}^2 \dot{Z}_{k0}, \\ \dot{S}_k' &= I_k^2 \dot{Z}_k, \\ \Delta\dot{S}_k &= \Delta I_k^2 \Delta\dot{Z}_k, \; k = \overline{1,3}, \end{aligned} \tag{15}$$

where $\dot{Z}_{k0}$, $\dot{Z}_k$ are the impedances of the virtual phases shown in Fig. 2a and Fig. 2b, respectively; $\Delta\dot{Z}_k$ is the impedance of load of the unauthorized consumer connected to the k-th phase. Estimates of the unknown quantities included in relations (15) will be determined further on.

The corresponding admittances are determined using the following equations:

$$\begin{aligned} \dot{Y}_{k0} &= \sum_{\nu=1}^{n} \dot{Y}_{k\nu}, \\ \dot{Y}_k &= \dot{Y}_{k0} + \Delta\dot{Y}_k, \\ \dot{Y}_{k\nu} &= \frac{1}{\dot{Z}_{k\nu}}, \quad \Delta\dot{Y}_k = \frac{1}{\Delta\dot{Z}_k}. \end{aligned} \tag{16}$$

Now, given (15) and (16), relations (14) can be written in the form:

$$I_{k0}^2 \dot{Z}_{k0} + \frac{\Delta I_k^2}{\Delta\dot{Y}_k} = \frac{I_k^2}{\dot{Y}_{k0} + \Delta\dot{Y}_k}, \; k = \overline{1,3}.$$

After simple transformations, given (15), they are equivalent to the following equalities:

$$\dot{S}_k^a \left(\dot{Y}_{k0} + \Delta\dot{Y}_k\right)\Delta\dot{Y}_k + \Delta I_k^2 \left(\dot{Y}_{k0} + \Delta\dot{Y}_k\right) = I_k^2 \Delta\dot{Y}_k, k = \overline{1,3},$$

By transforming these relations, we can derive quadratic equations with respect to unknown admittances $\Delta\dot{Y}_k$ of loads of unauthorized consumers:

$$a_k \left(\Delta\dot{Y}_k\right)^2 + b_k \Delta\dot{Y}_k + c_k = 0, \; k = \overline{1,3}, \tag{17}$$

where the coefficients a_k, b_k, c_k are determined by the equations:

$$\begin{aligned} a_k &= \dot{S}_k^a, \\ b_k &= \dot{S}_k^a \dot{Y}_k^0 + \Delta I_k^2 - I_k^2, \\ c_k &= \Delta I_k^2 \dot{Y}_k^0. \end{aligned}$$

To solve the quadratic equations (17), we can use the well-known Vieta's formula:

$$\Delta\dot{Y}_k = \frac{-b_k \pm \sqrt{b_k^2 - 4a_k c_k}}{2a_k}, \; k = \overline{1,3}. \tag{18}$$

Given (15), the coefficients b_k and the products of a_k and c_k are real values, i.e.,

$$\begin{aligned} b_k &= I_{k0}^2 + \Delta I_k^2 - I_k^2, \\ a_k c_k &= I_{k0}^2 \Delta I_k^2, \; k = \overline{1,3}, \end{aligned}$$

which simplifies the process of solving equations (17).

As a result, the required load impedances $\Delta\dot{Z}_k$ of unauthorized consumers are calculated as follows:

$$\Delta\dot{Z}_k = \frac{1}{\Delta\dot{Y}_k}, \; k = \overline{1,3}. \tag{19}$$

Then the required values of power losses $\Delta\dot{S}_k$, caused by electricity thefts in the phases of the physical network, given (8), are calculated by the equations:

$$\Delta\dot{S}_k = \frac{\Delta I_k^2}{\Delta\dot{Y}_k}, \; k = \overline{1,3}, \tag{20}$$

where the effective values of currents of unauthorized consumers ΔI_k will be determined below.

Now, based on the balance relations (1), we determine the values of technical power losses $\dot{S}_k^T$ in the phases of the physical network:

$$\dot{S}_k^T(\xi) = \dot{S}_k(\xi) - \dot{S}_k^a(\xi) - \Delta\dot{S}_k, \; k = \overline{1,3}, \tag{21}$$

where $\dot{S}_k(\xi)$ is the complex power consumed by the k-th phase of the physical network at the time $t = t_\xi$, which is calculated from the data of the main three-phase electricity meter by equations (2).

Let us assume that data from ASERM meters are collected at discrete moments of time $t = t_\xi$, $t = t_{\xi+1}$, $t = t_{\xi+2}$, …, $t = t_{\xi+m}$. Then the above computational procedure based on these data enables automated real-time control and monitoring of power losses in PDSs.

VI. Estimation of Desired Input Phase Currents of the Virtual Network

To calculate the effective values of the desired input phase currents I_{k0} in VN$_1$, the first relations of system (15) are written in the form:

$$\dot{S}_k^a = \frac{I_{k0}^2}{\dot{Y}_{k0}},\ k = \overline{1,3}. \qquad (22)$$

Next, the complex quantities $\dot{S}_k^a$ and $\dot{Y}_{k0}$ are expressed in their exponential form:

$$\dot{S}_k^a = S_k^a e^{j\varphi_k^a}, \qquad (23)$$

$$\dot{Y}_{k0} = Y_{k0} e^{-j\varphi_k^a},\ k = \overline{1,3},$$

where S_k^a, Y_{k0}, φ_k^a are the magnitudes and phases of the complex quantities $\dot{S}_k^a$ and $\dot{Y}_{k0}$, respectively.

Relations (22), given (23), have the form:

$$S_k^a e^{j\varphi_k^a} = \frac{I_{k0}^2}{Y_{k0}} e^{j\varphi_k^a},\ k = \overline{1,3}. \qquad (24)$$

Based on equalities (24), we obtain the required numerical values of the desired input phase currents I_{k0} of the virtual network, since S_k^a and Y_{k0} are known quantities:

$$I_{k0} = \sqrt{S_k^a Y_k^a} = \sqrt{\frac{S_k^a}{Z_k^a}},\ k = \overline{1,3}. \qquad (25)$$

VII. Estimation of Currents of Unauthorized Consumers

To estimate the currents of unauthorized consumers, relations (7) are used to write the expressions for complex currents $\Delta\dot{I}_k$ of unauthorized consumers:

$$\Delta\dot{I}_k = \dot{I}_k - \dot{I}_{k0},\ k = \overline{1,3}. \qquad (26)$$

Provisionally, these currents will be expressed in an exponential form:

$$\dot{I}_k = I_k e^{j(\beta_k + \alpha_k)},$$

$$\dot{I}_{k0} = I_{k0} e^{j\left(\beta_k + \alpha_k^0\right)},$$

where I_k, I_{k0}, α_k, α_k^0 are the effective values and phase shifts of the corresponding complex currents.

The following expressions can be written for the squares of the effective values of the unknown currents ΔI_k of unauthorized consumers:

$$\Delta I_k^2 = \left(\dot{I}_k - \dot{I}_{k0}\right)\left(\dot{I}_k - \dot{I}_{k0}\right)^* = \left(\dot{I}_k - \dot{I}_{k0}\right)\left(\dot{I}_k^* - \dot{I}_{k0}^*\right) =$$
$$= \dot{I}_k \dot{I}_k^* + \dot{I}_{k0}\dot{I}_{k0}^* - \left(\dot{I}_k \dot{I}_{k0}^* + \dot{I}_{k0}\dot{I}_k^*\right), \quad k = \overline{1,3}.$$

The components of these expressions are determined by the equations:

$$\dot{I}_k \dot{I}_k^* = I_k^2,\ \dot{I}_{k0}\dot{I}_{k0}^* = I_{k0}^2,$$

$$\dot{I}_k \dot{I}_{k0}^* + \dot{I}_k^* \dot{I}_{k0} = I_k I_{k0}\left(e^{j\left(\alpha_k - \alpha_k^0\right)} + e^{-j\left(\alpha_k - \alpha_k^0\right)}\right) =$$
$$= 2 I_k I_{k0} \cos\left(\alpha_k - \alpha_k^0\right).$$

Hence, we obtain expressions for the squares of the sought currents ΔI_k^2:

$$\Delta I_k^2 = I_k^2 + I_{k0}^2 - 2 I_k I_{k0} \cos\left(\alpha_k - \alpha_k^0\right), \quad k = \overline{1,3}.$$

Similarly, given (12), we obtain expressions for the squares of the currents I_k^2 and I_{k0}^2:

$$I_k^2 = I_{k0}^2 + \Delta I_k^2 + 2 I_{k0} \Delta I_k \cos\left(\alpha_k^0 - \Delta\alpha_k\right),$$

$$I_{k0}^2 = I_k^2 + \Delta I_k^2 - 2 I_k \Delta I_k \cos\left(\alpha_k - \Delta\alpha_k\right),\ k = \overline{1,3}.$$

Next, we introduce the following phase shift differences:

$$\eta_k = \alpha_k^0 - \Delta\alpha_k, \quad \lambda_k = \alpha_k - \Delta\alpha_k,$$
$$\theta_k = \lambda_k - \eta_k, \quad k = \overline{1,3}.$$

where $\alpha_k - \alpha_k^0 = \lambda_k - \eta_k$.

Based on the expressions obtained for the squares of the currents ΔI_k^2, I_k^2, and I_{k0}^2, we can write the following equations:

$$\begin{aligned} &\Delta I_k^2 - I_k^2 - I_{k0}^2 + 2 I_k I_{k0} \cos\theta_k = 0, \\ &I_k^2 - I_{k0}^2 - \Delta I_k^2 - 2\Delta I_k I_{k0} \cos\eta_k = 0, \\ &I_{k0}^2 - I_k^2 - \Delta I_k^2 + 2\Delta I_k I_k \cos\lambda_k = 0. \end{aligned} \qquad (27)$$

The entire set of relations (27) is a system of algebraic equations, which includes three equations with three unknown quantities ΔI_k, η_k and λ_k. The analysis of relations (27) indicates that the sought currents ΔI_k are determined by solving the following equations:

$$\Delta I_k^4 + e_k \Delta I_k^2 + r_k^2 +$$
$$+ q_k \sqrt{\Delta I_k^8 + m_k \Delta I_k^6 + d_k \Delta I_k^4 + s_k \Delta I_k^2 + r_k^4} = 0, \qquad (28)$$
$$k = \overline{1,3}.$$

where their coefficients are known quantities determined by the equations:

$$e_k = -2 f_k,\ f_k = I_k^2 + I_{k0}^2,$$
$$r_k = I_k^2 - I_{k0}^2,\ m_k = -4 f_k,$$
$$d_k = 16 I_k^2 I_{k0}^2 + 6 r_k^2, s_k = -4 f_k r_k^2.$$

The sign function q_k is selected as follows:

$$q_k = \begin{cases} +1, \text{ if } \operatorname{sign}\left(\Delta I_k^4 + e_k \Delta I_k^2 + r_k^2\right) < 0, \\ -1, \text{ if } \operatorname{sign}\left(\Delta I_k^4 + e_k \Delta I_k^2 + r_k^2\right) > 0. \end{cases}$$

Solving the algebraic equation (28) for a given k is not particularly difficult if numerical methods are used [18–20], since its approximate solution ΔI_{k0} can be obtained on the basis of relation (26), i.e., $\Delta I_{k0} = I_k - I_{k0}$.

VIII. GENERAL ALGORITHM FOR POWER LOSS DETECTION IN THE NETWORK

Based on the obtained results, we can formulate the following high-level algorithm for power loss detection and monitoring in distribution systems.

1. Poll cyclically the main three-phase electricity meter and customers' electricity meters of ASERM at a discrete instant of time $t = t_\xi$.
2. Enter the received information into the ASERM database and build vectors $I' = \left[I'_1, I'_2, I'_3\right]$.
3. Calculate impedances $\dot{Z}_{kv}$ and admittances $\dot{Y}_{kv}$, as well as complex powers $\dot{S}_{kv}$ consumed by the network customers, using equations (9)–(11), respectively.
4. Estimate total powers $\dot{S}_k^a$ and impedances $\dot{Z}_k^a$ using equations (3) and (24), respectively.
5. Determine desired input phase currents I_{k0}, $k = \overline{1,3}$, using equations (25).
6. Find the values of uncontrolled currents ΔI_k, $k = \overline{1,3}$, by solving the algebraic equations (28).
7. Check the current state of the PDS by checking conditions (13).
8. When conditions (13) are satisfied, there are no ETs in the network. Go to Step 1. Otherwise, go to Step 9.
9. Calculate admittances $\Delta\dot{Y}_k$ by solving equations (17).
10. Estimate non-technical $\Delta\dot{S}_k$ and technical $\dot{S}_k^T$ ($k = \overline{1,3}$) power losses in the PDS using equations (20) and (21), respectively, at time $t = t_\xi$.
11. Go to step 1 and repeat the computational procedure for successive instants of time $t = t_{\xi+m}$, where $m = 1, 2, 3, \ldots$

IX. CONCLUSION

The operating practices of modern automated systems for electricity revenue metering, which are used to control and measure electric power in 0.4 kV power distribution systems, demonstrate a noticeable lack of digital technologies. That means that they are unable to provide real-time detection and monitoring of power losses with separate detection and estimation of technical and non-technical losses. This paper introduces a novel approach to address this issue, utilizing a virtual model of the power distribution system. The virtual model describes the desired state of the power distribution system in the absence of external random factors, which, in particular, include electricity thefts. This model utilizes metering data received via communication links from electricity meters installed in the transformer substation and at network customers. The systems of algebraic equations are obtained to describe functional relations between variables and parameters of the virtual model of a three-phase network. We proposed the techniques to solve these equations and a generalized algorithm to detect and monitor technical and non-technical power losses in power distribution systems. The findings of this research will be used to advance the development of algorithmic and specialized software for the subsystem of real-time power loss monitoring in PDSs as part of the ASERM.

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